Symbolic Methods for Verifying Software

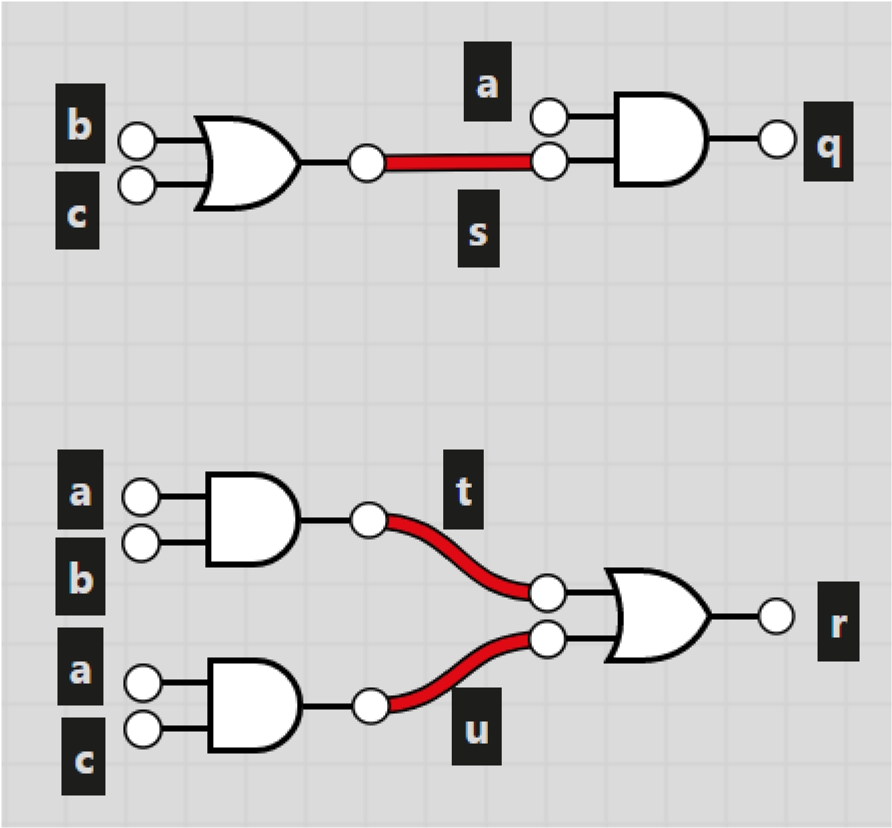
V&T

Roderick Bloem

IAIK – Graz University of Technology
Roderick.Bloem@iaik.tugraz.at

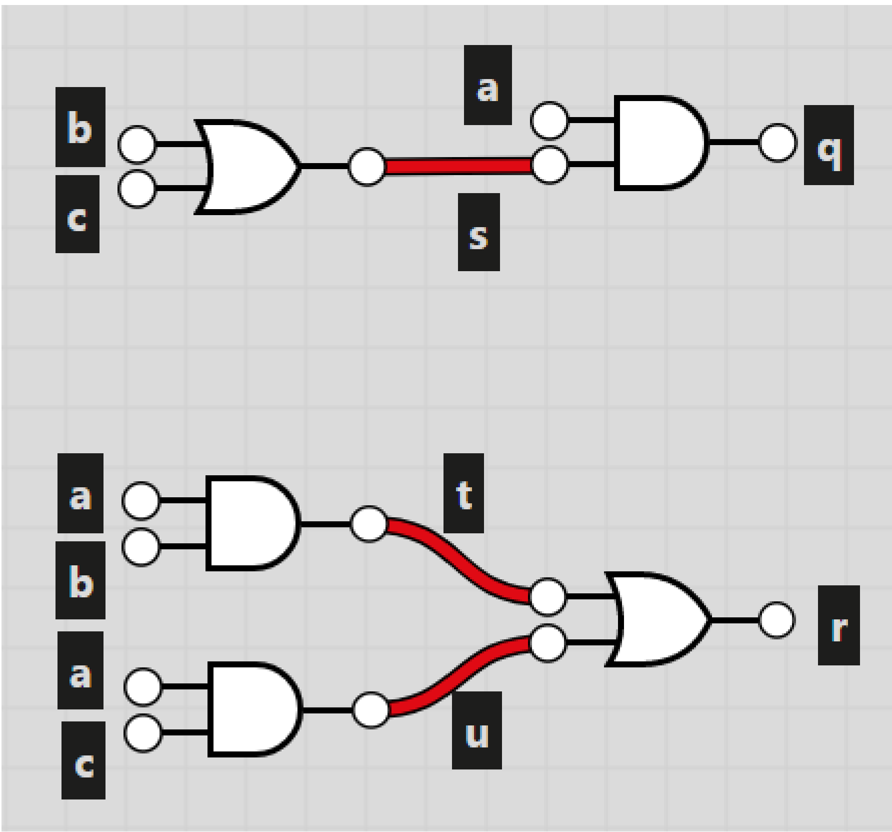
Note to me: see code under Code/Cbmc

Circuit Equivalence



https://logic.ly/demo

Circuit Equivalence



$$\Phi = (s \leftrightarrow b \vee c) \wedge (q \leftrightarrow a \wedge s).$$

$$\Psi = (t \leftrightarrow a \wedge b) \wedge u \leftrightarrow (a \wedge c) \wedge (r \leftrightarrow t \vee u).$$

Circuits are different iff following is satisfiable

$$\Phi \wedge \Psi \wedge (q \neq r)$$

Z3

```
(declare-const a Bool)
(declare-const b Bool)
(declare-const c Bool)
(declare-const p Bool)
(declare-const q Bool)
(declare-const r Bool)
(declare-const s Bool)
(declare-const t Bool)
(declare-const u Bool)

(assert (= s (or b c)))
(assert (= q (and a s)))

(assert (= t (and a b)))
(assert (= u (and a c)))
(assert (= r (or t u)))

(assert (not (= q r)))

(check-sat)
(get-model)
```

https://rise4fun.com/Z3 or

https://compsys-tools.ens-lyon.fr/z3/index.php

Circuit Equivalence

- *Combinational* circuits (no memory elements): Use **Tseitin transformation**
  - Give each wire a name
  - Use standard formula for each gate
  - conjoin formulas
- Note: linear construction
- More complicated for *sequential* circuits (with memory)
  - model checking using a SAT solver, interpolation

The Following is a Bad Idea

Don't do following (exponential blowup)

$z = x \vee y$ [substitute x and y]

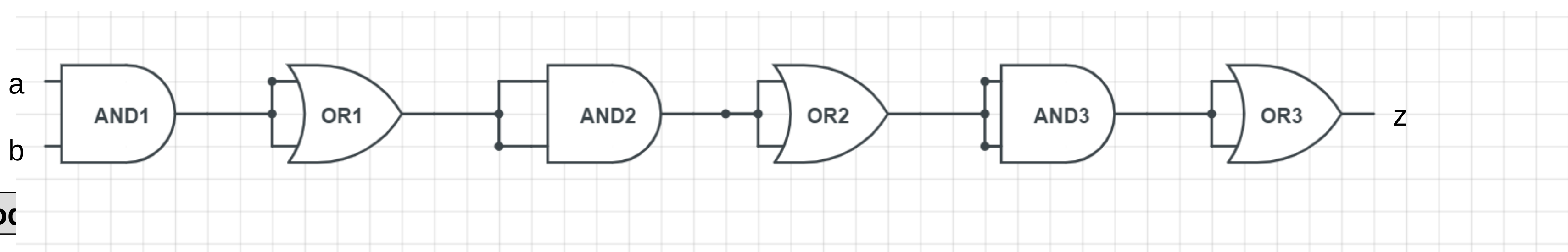
$z = (v \wedge w) \vee (v \wedge w)$ [substitute v, w]

$z = \big((t \vee u) \wedge (t \vee u)\big) \vee ((t \vee u) \wedge (t \vee u))$ [now t,u]

etc etc

Happens whenever circuit has reconvergence (reuse of values)

u w



Verification Condition

Given a Program P, a **verification condition** is a formula ϕ such that

(ϕ satisfiable) implies (P buggy).

For the circuit example, the verification condition is $\Phi \wedge \Psi \wedge q \neq r$

From Circuits to Software

Find out if the assertion can be violated

```
Boolean a, b;
if(a){
  if(!b)
    assert(false);
}
```

← How do I get here?

ϕ?

Assertion reached iff ϕ satisfiable.

From Circuits to Software

Find out if the assertion can be violated

```
Boolean a, b;
if(a){
  if(!b)
    assert(false);
}
```

How do I get here?

$$\phi = a \land \neg b$$

Assertion reached iff ϕ satisfiable.

Satisfying assignment = input to reach assertion

Adding Assignments

```
Boolean a, b;
if(a){
  a = (a&&b);
  if(!a)
    assert(false);
}
```

```
Boolean a0, b0, a1;
if(a0){
  a1 = (a0&&b0);
  if(!a1)
    assert(false);
}
```

Single Static Assignment (SSA)

Let $\phi =$

Assertion reached iff ϕ satisfiable

Adding Assignments

```
Boolean a, b;
if(a){
  a = (a&&b);
  if(!a)
    assert(false);
}
```

```
Boolean a0, b0, a1;
if(a0){
  a1 = (a0&&b0);
  if(!a1)
    assert(false);
}
```

Single Static Assignment (SSA)

Let $\phi = \text{a0} \wedge (a1 \leftrightarrow a0 \wedge b0) \wedge \neg a1$.
Assertion reached iff ϕ satisfiable

Adding Arithmetic

```
int a, b, c;
if(a != 0){
  c = (a + b);
  if(c > 0)
    assert(false);
}
```

Let's pretend ints have four bits

a != 0

Adding Arithmetic

```
int a, b, c;
if(a != 0){
  c = (a + b);
  if(c > 0)
    assert(false);
}
```

Let's pretend ints have four bits

c > 0

Adding Arithmetic

```
int a, b, c;
if(a != 0){
  c = (a + b);
  if(c > 0)
    assert(false);
}
```

Let's pretends `int`s are 4 bits: a_3, a_2, a_1, a_0

`(a != 0)` becomes $a_0 \vee a_1 \vee a_2 \vee a_3$

`(c>0)` becomes $\neg c_3 \wedge (c_2 \vee c_1 \vee c_0)$

What about addition?

Adding Arithmetic

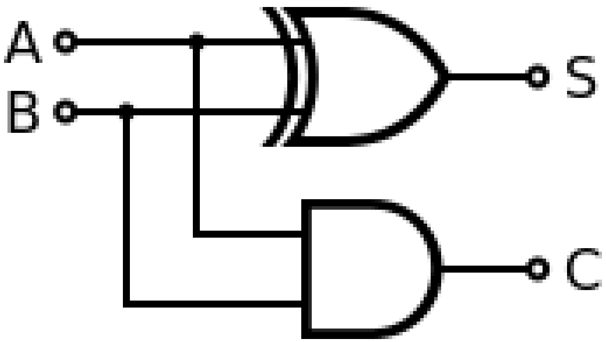
```
int a, b, c;
if(a != 0){
  c = (a + b);
  if(c > 0)
    assert(false);
}
```

Let‘s pretend ints have four bits

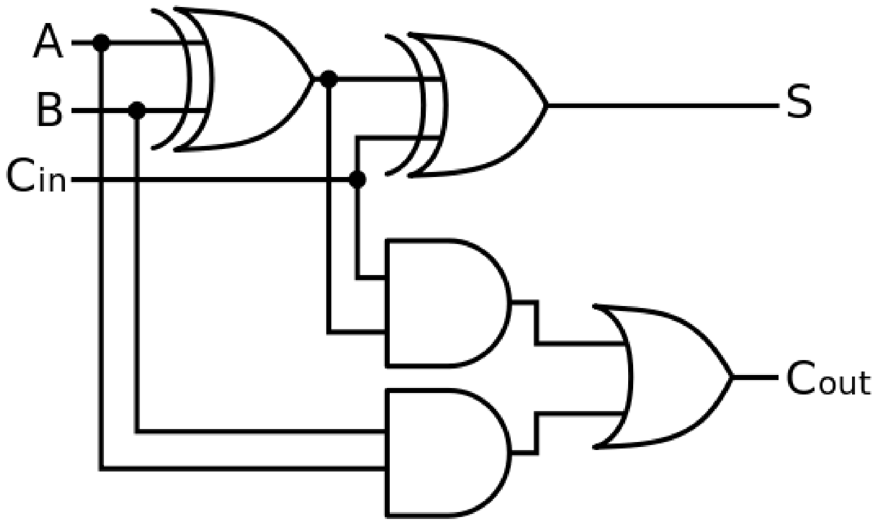
c = a + b

One-Bit Adder

Half Adder

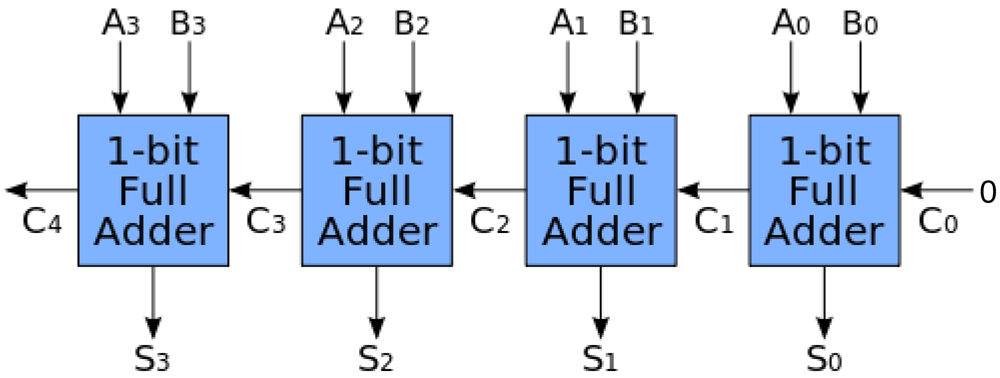


Full Adder



Halfadder: Wikipedia, public domain; full adder: **Colin M.L. Burnett**, GDFL

4-bit Adder



Write formula

$\phi(a_3, a_2, a_1, a_0, b_3, b_2, b_1, b_0, s_3, s_2, s_1, s_0)$ such that

$\phi(a, b, s)$ is true iff $s = a + b$.

Note: there are extra variable in ϕ that don't bother us (why not?)

Software

```
int a, b, c;
if(a != 0){
  c = (a + b);
  if(c > 0)
    assert(false);
}
```

Let's pretends `int`s are 4 bits: a_3, a_2, a_1, a_0

$$\psi(a,b,c) =$$

$$(a_0 \vee a_1 \vee a_2 \vee a_3) \wedge$$ `a != 0`

$$\phi(a,b,c) \wedge$$ `c = a +b`

$$\neg c_3 \wedge (c_2 \vee c_1 \vee c_0)$$ `c > 0`

ψ satisfiable iff assertion reachable.

Summarizing

We know how to represent a single path in a formula

From now on, I will use arithmetic in my functions

How do we deal with multiple paths and conditions? Two options:

1. Bounded Model Checking
2. Concolic Testing

Bounded Model Checking

- Create a formula that says a bug exist, give to SMT solver.
- Formula: *Is there a path of length* $\leq$ *k to a bug?*

Tool: CBMC

From Path to Program: BMC

Program

```
int a, b, c;
if(c > 0){
   assert(c < a);
else
   assert (c > a);
```

Formula

$\phi =$

ϕ is true iff the program contains a bug.

idea: represent all paths in a formula

From Path to Program: BMC

Program

```
int a, b, c;
if(c > 0){
  assert(c < a);
else
  assert (c > a);
```

Formula

Path condition

$$\phi = (c > 0) \land \neg(c < a) \lor \neg(c > 0) \land \neg(c > a)$$

ϕ satisfiable iff program contains bug.

idea: represent all paths in one formula

Loop unrolling

Program

```
int a, b, as, bs;
b = b > 0 ? b : -b;
as = a;
bs = b;
while(b>0){
  a = a + 1;
  b = b - 1;
}
assert(a == as + bs);
```

Formula

BMC: Loop Unrolling

Program

```
int a, b, as, bs;
b = b > 0 ? b : -b;
as = a;
bs = b;
while(b>0){
   a = a + 1;
   b = b - 1;
}
assert(a == as + bs);
```

Program'(unroll 0)

```
int a, b, as, bs;
b = b > 0 ? b : -b;
as = a;
bs = b;
if(b>0){
   stop;
}
assert(a == as + bs);
```

print a warning unrolling not long enough!

BMC: Loop Unrolling

Program

```
int a, b, as, bs;
b = b > 0 ? b : -b;
as = a;
bs = b;
while(b>0){
  a = a + 1;
  b = b - 1;
}
assert(a == as + bs);
```

Program'(unroll 1)

```
int a, b, as, bs;
b = b > 0 ? b : -b;
as = a;
bs = b;
if(b>0){
  a = a + 1;
  b = b - 1;
  if(b>0) stop;
}
assert(a == as + bs);
```

BMC: Loop Unrolling

Program

```
int a,b,as,bs;
b = b>0 ? b : -b;
as = a;
bs = b;
while(b>0){
  a = a + 1;
  b = b - 1;
}
assert(a==as+bs);
```

Program'(1)

```
int a,b,as,bs;
b = b>0 ? b : -b;
as = a;
bs = b;
if(b>0){
  a = a + 1;
  b = b - 1;
  if(b>0) stop;
}
assert(a==as+bs);
```

Program''(2)

```
int a,b,as,bs;
b = b>0 ? b : -b;
as = a;
bs = b;
if(b>0){
  a = a + 1;
  b = b - 1;
  if(b>0){
    a = a + 1;
    b = b - 1
    if(b>0) stop;
  }
}
assert(a==as+bs);
```

BMC: Loop Unrolling

Program'

```
int a, b, as, bs;
b = b>0 ? b : -b;
as = a;
bs = b;
if(b>0){
   a = a + 1;
   b = b - 1;
   if(b>0) stop;
}
assert(a == as + bs);
```

Program' (SSA)

```
int a, b, as, bs;
b0 = b>0 ? b : -b;
as = a;
bs = b0;
if(b0>0){
   a1 = a + 1;
   b1 = b0 - 1;
   if(b1>0) stop;
} else {
   a1 = a; b1 = b0;
}
assert(a1 == as + bs);
```

Verification Condition

```
int a, b, as, bs;
b0 = b>0 ? b : -b;
as = a;
bs = b;
if(b0>0){
  a1 = a + 1;
  b1 = b0 - 1;
  if(b1>0) stop;
} else {
  a1 = a; b1 = b0;
}
assert(a1 == as + bs);
```

Finding assertion violation

$\phi_1 = (b > 0 \land b_0 = b \lor b \leq 0 \land b_0 = -b) \land as = a \land bs = b_0 \land b_0 \leq 0 \land a_1 = a \land b_1 = b_0$

$\phi_2 = (b > 0 \land b_0 = b \lor b \leq 0 \land b_0 = -b) \land as = a \land bs = b_0 \land b > 0 \land a_1 = a + 1 \land b_1 = b_0 + 1$

Verification condition: $\phi = (\phi_1 \lor \phi_2) \land a_1 \neq as + bs$

Have we unrolled enough?

Let

$\psi = (b > 0 \land b_0 = b \lor b \leq 0 \land b_0 = -b) \land as = a \land bs = b_0 \land b_0 > 0 \land a_1 = a + 1 \land b_1 = b_0 - 1 \land b_1 > 0$

If ψ satisfiable, verification incomplete: unroll loop further!

Formulas

Circumstances: assignments to initial variables (and other variables along a path)

Path condition: Under which circumstances can I get to a given point in the program?

Verification condition: Under which circumstances does the program fail?

Unrolling condition: Under which circumstances does the program continue beyond unrolling bound?

$\phi_1 = (b > 0 \wedge b_0 = b \vee b \leq 0 \wedge b_0 = -b) \wedge as$
$= a \wedge bs = b_0 \wedge b_0 \leq 0 \wedge a_1 = a \wedge b_1 = b_0$

$\phi_2 = (b > 0 \wedge b_0 = b \vee b \leq 0 \wedge b_0 = -b) \wedge as$
$= a \wedge bs = b_0 \wedge b > 0 \wedge a_1 = a + 1 \wedge b_1$
$= b_0 + 1$

Path condition for *: $\phi_1 \vee \phi_2$

Verification condition: $(\phi_1 \vee \phi_2) \wedge a1 \neq as + bs$

Unrolling condition: $\psi = (b > 0 \wedge b_0 = b \vee b \leq 0 \wedge b_0 = -b) \wedge as = a \wedge bs = b_0 \wedge b_0 > 0 \wedge a_1 = a + 1 \wedge b_1 = b_0 - 1 \wedge b_1 > 0$

```
int a, b, as, bs;
b0 = b>0 ? b : -b;
as = a;
bs = b;
if(b0>0){
  a1 = a + 1;
  b1 = b0 - 1;
  if(b1>0) stop;
} else {
  a1 = a; b1 = b0;
}
assert(a1 == as + bs);
```

ψ ϕ_1 ϕ_2

Algorithm

```
k = 0
while(true)
        unroll program to depth k, use SSA
        ϕ = verification condition
        ψ = unrolling condition
        if(ϕ SAT) halt("found a bug")
        if(ψ UNSAT) halt("no bug exists")
        k++
}
```

Note: This is for one loop.
For multiple loops:

- bug exists if any verification condition is satisfiable
- program is correct if all unrolling conditions are unsatisfiable.

Formulas

Path condition: Satisfiable iff point reachable

Verification condition: Satisfiable implies program buggy

Unrolling condition: Satisfiable iff program should be unrolled more

Loop unrolling

Check for bugs that occur when the loops are unrolled *k* times, for some *k.*

Good:

- Find **all** bugs for any input up to that depth

Bad:

- Expressions quickly become complicated; you will not go *deep* into a program

What if we want to test deeply?

Concolic Testing

- Idea: combine random testing with symbolic execution. Then, systematically look for inputs that take a different path.
- Formula: *Can this path lead to a bug for some input?*

Tools: DART, CUTE
(see also KLEE for symbolic execution)

Concolic Testing Example

- values = random input
- while(true)
  - Execute program with concrete inputs and symbolically at the same time.
    - Concrete values determine path
    - Build path condition as you go
  - Negate part of path condition to obtain different path
  - Give to solver to obtain new values

Note: will treat `assert(c)` as `if(c)`
`assert(false)`

Effect: we can ask for assertion violations

Path Condition

Path condition: formula that states how to get to a given location.

assertion reached with path condition ?

```
int h(int x, int y) {
  if (x == y)
    if (x*x == 16)
      abort(); /*error*/
     else
       assert(y==4);
  return 0;
}
```

Concolic Testing Example

1. Start with random input
2. Execute program with concrete **and** symbolic inputs. Concrete inputs determine path
3. Check for bug on path
4. Negate part of condition to obtain different path
5. Obtain new values from solver
6. Repeat

```
int h(int x, int y) {
  if (x == y)
    if (x*x == 16)
      assert(y==4);
  return 0;
}
```

1. Call `h(12,88)`

2. h takes else branch h. Path condition: $\phi_1 = (x \neq y)$
3. There is no assertion on the path, so no bug
4. $\neg\phi_1 = (x = y)$
5. Solver gives, e.g., x = 42, y= 42

2. new call: h(42,42). Program takes then branch and else branch. Path condition: $\phi_2 = (x = y) \wedge (x \cdot x \neq 16)$.
3. No assertion -> no bug
4. Obtain $\phi = \phi_1 \vee \neg\phi_2 = (x = y) \wedge (x \cdot x = 16)$
5. Obtain an assignment for $\neg\phi$: x=4, y =4.

2. New call: h(4,4).
3. Assertion is reached but not violated. Now vheck $(x = y) \wedge (x \cdot x = 16) \wedge (y \neq 4)$
BUG: x==-4, y == -4!

Concolic Testing

In which order do we change conditions?

- Any search order we want.
- Example: always negate last part of condition $\rightarrow$ DFS

1. Start with random input
2. Execute program with concrete and symbolic inputs. Concrete inputs determine path
3. Check for bug on path
4. Negate part of condition to obtain different path
5. Obtain new values from solver
6. Repeat

Dealing with Memory

Random pointers make little sense – prefer NULL pointers, or allocated structs.

Dealing with Memory

```
typedef struct cell{
  int v;
  struct cell *next;
} cell;

int f(int v){ return 2*v+1;
   }

int testme(cell *p, int x){
  if(x > 0)
    if(p != NULL)
      if(f(x) == p->v)
        if(p->next == p)
          ERROR;
  return 0;
}
```

Dealing with Memory

```
typedef struct cell{
  int v;
  struct cell *next;
} cell;

int f(int v){ return 2*v+1;
   }

int testme(cell *p, int x){
  if(x > 0)
    if(p != NULL)
      if(f(x) == p->v)
        if(p->next == p)
          ERROR;
  return 0;
}
```

Start: x=236; p = NULL

path: x>0; p==NULL.

Solve x>0 && p!=NULL

x=236, p->[634,NULL]

path: x>0; p!=NULL; 2x+1 != p->v

solve x>0 && p != NULL && 2x+1==p->v

x=1; p->[3,NULL]

path: x>0; p!=NULL; 2x+1 == p->v; p->next!=p

solve x>0 && p != NULL && 2x+1==p->v && p->next==p

x=1; p->[3,p]

ERROR reached

Conclusions

- Symbolic representation of programs
- Systematic search for all bad behavior

BMC tries all paths simultaneously.

- Query: Are there inputs such that some path of length k leads to a bug
- Like breadth-first search: wide and shallow

Concolic tries one path at a time

- Query: Are there inputs such that this path leads to a bug
- Like depth-first search: deep and narrow

Literature

- P. Godefroid, N. Klarlund, and K. Sen, DART: Directed Automated Random Testing*, Proc. Programming Language Design and Implementation,* 2005
- K. Sen, D. Marinov, and G. Agha, CUTE: A Concolic Unite Testing Engine for C, Proc. *European Software Engineering Conference and ACM SIGSOFT Symposium on the Foundations of Software Engineering, 2005*