

Deduction Rules

Propositional Logic

	Introduction	Elimination
\wedge	$\frac{\varphi \quad \psi}{\varphi \wedge \psi} \wedge i$	$\frac{\varphi \wedge \psi}{\varphi} \wedge e1 \quad \frac{\varphi \wedge \psi}{\psi} \wedge e2$
\vee	$\frac{\varphi}{\varphi \vee \psi} \vee i1 \quad \frac{\psi}{\psi \vee \varphi} \vee i2$	$\frac{\varphi \vee \psi \quad \begin{array}{ l} \varphi \text{ ass.} \\ \vdots \\ \chi \end{array} \quad \begin{array}{ l} \psi \text{ ass.} \\ \vdots \\ \chi \end{array}}{\chi} \vee e$
\rightarrow	$\frac{\begin{array}{ l} \varphi \text{ ass.} \\ \vdots \\ \psi \end{array}}{\varphi \rightarrow \psi} \rightarrow i$	$\frac{\varphi \quad \varphi \rightarrow \psi}{\psi} \rightarrow e$
\neg	$\frac{\begin{array}{ l} \varphi \text{ ass.} \\ \vdots \\ \perp \end{array}}{\neg \varphi} \neg i$	$\frac{\varphi \quad \neg \varphi}{\perp} \neg e$
\perp	no rule	$\frac{\perp}{\varphi} \perp e$
$\neg\neg$	$\frac{\varphi}{\neg\neg\varphi} \neg\neg i$	$\frac{\neg\neg\varphi}{\varphi} \neg\neg e$

Derived Rules

$\frac{\varphi \vee \neg\varphi}{\text{LEM}}$	$\frac{\begin{array}{ l} \neg\varphi \text{ ass.} \\ \vdots \\ \perp \end{array}}{\varphi} \text{PBC}$	$\frac{\varphi \rightarrow \psi \quad \neg\psi}{\neg\varphi} \text{MT}$
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Predicate Logic

	Introduction	Elimination
=	$\frac{}{t=t} =i$	$\frac{t_1 = t_2 \quad \Phi [t_1/x]}{\Phi [t_2/x]} =e$
\forall	$\frac{\boxed{\begin{array}{l} x_0 \\ \vdots \\ \Phi [x_0/x] \end{array}} \quad x_0 \text{ fresh}}{\forall x \Phi} \forall i$	$\frac{\forall x \Phi}{\Phi [t/x]} \forall e$
\exists	$\frac{\Phi [t/x]}{\exists x \Phi} \exists i$	$\frac{\exists x \Phi \quad \boxed{\begin{array}{l} x_0 \\ \Phi [x_0/x] \text{ ass.} \\ \vdots \\ \chi \end{array}} \quad x_0 \text{ fresh}}{\chi} \exists e$