

Hardware Acceleration Opportunities in Homomorphic Encryption

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Privacy-Preserving Outsourcing of Computation



Diabetic Retinopathy [Chao et al., 2019]

User wants to compute *foo(data*) in the cloud without loosing privacy.

Fully Homomorphic Encryption (FHE)

FHE enables computation on encrypted data



Dec() gives foo(data)

Cloud *homomorphically* evaluates *foo*()

Tutorial outline

- 1. FHE concepts
- 2. Parallel processing opportunities in FHE (from high-level)
- 3. Hardware architecture design challenges and methods
- 4. Results

Definition: Homomorphic Encryption

An encryption scheme $Enc(\cdot, \cdot)$ is homomorphic for an operation \Box on the message space iff

$$Enc(m_1 \square m_2, k_E) = Enc(m_1, k_E) \circ Enc(m_2, k_E)$$

with o operation on the ciphertext.

- If \Box = + then Enc(\cdot , \cdot) is additively homomorphic.
- If $\Box = \times$ then Enc(\cdot , \cdot) is multiplicatively homomorphic.

Example: Textbook RSA is multiplicatively homomorphic

• You have encryption of two messages m₁ and m₂ where

 $c_1 = m_1^e \mod N$ $c_2 = m_2^e \mod N$

• By multiplying c_1 and c_2 you get $c_3 = c_1 \cdot c_2 = (m_1 \cdot m_2)^e \mod N$

• Hence, c_3 is encryption of $m_1 \cdot m_2$

Can we get 'Additive & Multiplicative' Homomorphic Encryption?

Popular constructions of FHE use augmented Ring-LWE public-key encryption

Recap -- Ring LWE Public-Key Encryption (PKE)

Encryption:

□ Input: $pk = (p_0, p_1)$, message m □ Output: $ct = (ct_0, ct_1)$



Recap -- Ring LWE Public-Key Encryption (PKE)



$$ct_0 + ct_1.s = m' = Enc(m) + (e.s' + e'' + e'.s)$$
$$= Enc(m) + e_{small}$$

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Equivalently,

$$\frac{\operatorname{ct}_0 + \operatorname{ct}_1 \cdot \mathrm{s}}{\mathrm{q}/2} \mod 2 = \mathrm{m}$$

Ring-LWE PKE – Written with different symbols

Let scale factor $\Delta = q/t$ and t be plaintext modulus, e.g., t = 2. Scalars are in red. All polynomials are in blue.



Ring-LWE PKE shows Homomorphism



Now consider two ciphertexts $Ct_A = \{ct_{A0}, ct_{A1}\}$ and $Ct_B = \{ct_{B0}, ct_{B1}\}$

 $e_{A0}, e_{A1}, u_A \leftarrow error();$ $ct_{A0} = p_0 \cdot u_A + e_{A0} + \Delta \cdot m_A$ $ct_{A1} = p_1 \cdot u_A + e_{A1}$ $e_{B0}, e_{B1}, u_{B} \leftarrow error();$ $ct_{B0} = p_{0} \cdot u_{B} + e_{B0} + \Delta \cdot m_{B}$ $ct_{B1} = p_{1} \cdot u_{B} + e_{B1}$

Ring-LWE PKE: Additive Homomorphism





















The Biggest Problem in FHE



Polynomial size



FHE does lots of (large) polynomial arithmetic.

How to accelerate FHE?

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What makes acceleration of FHE very challenging?

- Lots of polynomial arithmetic operations
 - Large degree polynomial arithmetic
 - Long integer arithmetic
- Memory management
 - Ciphertexts could be several MBs
 - On-Chip memory is limited
 - Off-Chip data transfer is very slow

What makes acceleration of FHE very challenging?

- Lots of polynomial arithmetic operations
 - Large degree polynomial arithmetic
 - Long integer arithmetic This problem is solved using CRT
- Memory management
 - Ciphertexts could be several MBs
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Dealing with long-int coefficients using RNS

- 1. Take a modulus $Q = \prod_{i=1}^{L-1} q_i$ where q_i are coprime.
- 2. Use Residue Number System (RNS).



E.g., Parallel computation flow with CRT



... (1) Residue polynomial arithmetic layer



*RPAU stands for 'Residue Polynomial Arithmetic Unit'

... (2) Residue polynomial layer \iff CRT layer



Each thread in CRT layer combines *all threads* from previous layer.

... (3) Residue polynomial layer \iff CRT layer

Therefore, threads or RPAUs need to exchange data with each other.



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System-level view





How to multiply two very large polynomials?

- Schoolbook multiplication: $O(n^2)$
- Karatsuba multiplication: $O(n^{1.585})$
- Toom-Cook (generalization of Karatsuba)
- Fast Fourier Transform (FFT) multiplication: O(n log n)

FFT is the best choice

Asymptotic complexity plays its role.

NTT-based Polynomial Multiplication



NTT or Number Theoretic Transform is special FFT with integers.

Let's consider an application example.

Polynomial size n = 2^{15} And *log*(q_i) = 60

NTT and of a polynomial A[] Simplified NTT loops



NTT and Memory access

Simplified NTT loops


Simplified NTT loops



Simplified NTT loops



Simplified NTT loops



Simplified NTT loops



Can we speedup polynomial multiplication using several NTT cores in parallel?

Answer: Yes

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Is parallel NTT easy to implement?

Answer: Complexity of implementation increases with number of cores

Parallel NTT Challenge 1: Port limitation in BRAM or SRAM



Problem:

- One BRAM has only two ports.
- Each NTT core needs two ports

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Solution: Use BRAMs in parallel.

New problem: How to distribute data?

Parallel NTT Challenge 2: Memory access conflicts

Two or more cores try to read/write the same BRAM element. But BRAM has a limited number of ports to satisfy one core.



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Parallel NTT Challenge 3: Data routing

Core requires data from distant BRAM memory

- Long routing of data wires \rightarrow slow clock frequency



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Solution: There is no solution to this problem. Localizing read or write (not both) the read operation.

Compute Core-C * Bus Switching Matrix Data-write paths are . heavily pipelined. This paper localizes Compute Core-1 BRAM is exclusively read by only one Pipeline register core. Compute Coefficient of a polynomial * Core-0 * NTT Cores **BRAMs**

Mert et al. "Medha: Microcoded Hardware Accelerator for computing on Encrypted Data". TCHES 2023

System-level view: Main challenges



Next topic: Memory management

Memory organization and management



Common techniques

• Lots of on-chip memory (BRAM/SRAM) for storing operands

Memory organization and management



Common techniques

- Lots of on-chip memory (BRAM/SRAM) for storing operands
- Perform communication-computation parallelism using cache

Memory organization and management



Common techniques

- Lots of on-chip memory (BRAM/SRAM) for storing operands
- Perform communication-computation parallelism using cache
- High-bandwidth off-chip memory and with multiple channels

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 Implementation
- 4. Results

Implementations

There are two main tracks

- 1. True accelerator prototype in ASIC/FPGA
- 2. Simulation-based modelling of accelerator

Real HW prototypes: HEAWS^[1], HEAX^[2], CoFHEE^[3], Medha^[4]

Simulation-based works: F1^[5], BTS^[6], CraterLake^[7], ...

[1] Furkan Turan et al. HEAWS: an accelerator for homomorphic encryption on the amazon AWS FPGA. IEEE ToC, 2020.
 [2] M. Sadegh Riazi et al. HEAX: an architecture for computing on encrypted data. ASPLOS 2020.
 [3] Mohammed Nabeel et al. CoFHEE: A Co-processor for Fully Homomorphic Encryption Execution. DATE 2023.
 [4] Mert et al. Medha: Microcoded Hardware Accelerator for computing on Encrypted Data. CHES 2023.
 [5] Axel Feldmann et al. F1: A fast and programmable accelerator for fully homomorphic encryption. MICRO 2021.
 [6] Sangpyo Kim et al. BTS: An Accelerator for Bootstrappable Fully Homomorphic Encryption. ISCA 2022.
 [7] Samardzic et al. CraterLake: A Hardware Accelerator for Efficient Unbounded Computation on Encrypted Data. ISCA 2022.

Briefly talk about

Next, FHE accelerator



High level computation flow



... Residue polynomial arithmetic layer



*RPAU stands for Residue Polynomial Arithmetic Unit

... Residue polynomial layer \iff CRT layer



Each thread in CRT layer combines *all threads* from previous layer.

... Residue polynomial layer 🔶 CRT layer

Therefore, RPAUs need to exchange data with each other.



RPAU ()



Example RPAU. It uses 16 NTT butterfly cores and 4 coefficient-wise (dyadic) arithmetic cores. Polynomials are stored in 'Memory' made of BRAMs.

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Instruction Parallelism in RPAU ()

 $\tilde{d}_{0,j} \leftarrow \tilde{c}_{0,j} \star \tilde{c}_{0,j}'$ HE. Mult $ilde{d}_{1,j} \leftarrow ilde{c}_{0,j} \star ilde{c}_{1,j}' + ilde{c}_{1,j} \star ilde{c}_{0,j}'$ $\tilde{d}_{2,j} \leftarrow \tilde{c}_{1,j} \star \tilde{c}_{1,j}'$ $\{c_{0,i}'',c_{1,i}''\} \leftarrow 0$ HE. Relin $d_{2,i} \leftarrow \text{INTT}(d_{2,i})$ for i = 0 to L - 1 do Obtain $d_{2,i}$ from RPAU_i $r_{2,i} \leftarrow \texttt{Coeff.Reduce}(d_{2,i}, q_j)$ $\tilde{t} \leftarrow \operatorname{NTT}(r_{2,i})$ $c_{0,i}'' \gets c_{0,i}'' + \mathtt{KSK}_{0,i} \star \tilde{t}$ $c_{1,i}'' \leftarrow c_{1,i}'' + \texttt{KSK}_{1,i} \star \tilde{t}$ end for $(d_{0,i}, d_{1,i}) \leftarrow |c'' \cdot p^{-1}|$

Homomorphic multiplication & key-switching. (The most expensive operation)

Parallel execution of instructions



This reduces 40% cycle count

Placement of RPAUs

CRT requires combining the residues.

 \rightarrow Therefore, RPAUs need to communicate with each other

How to interconnect the RPAUs in large 3D FPGAs?



Large SLR FPGA

Large FPGAs are multi-die

- > The FPGA is split into four SLRs.
- Connected by a limited number of wires.



Large SLR FPGA – top view



There are a limited number of interconnects.

Large design cannot be spread arbitrarily across SLRs.

Xilinx Alveo U250 FPGA. This FPGA is 1000x larger than the FPGA used in this course.

- FPGA Constraints
 - > The FPGA is split into four SLRs.
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- Some operations require exchanging the residue polynomials between RPAUs
- Naïve solution: A "star-like" network





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Each RPAU has its own connections





- FPGA Constraints
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- Naïve solution: A "star-like" network

- Complicates the routing
- Large number of nets crossing the SLRs
- Reduces the clock frequency to around 50 MHz or less



- FPGA Constraints
 - > The FPGA is split into four SLRs.
 - Connected by a limited number of wires.
- Some operations require exchanging the residue polynomials between RPAUs
- Solution: A "ring" interconnection of RPAUs

- Only two neighbour RPAUs are connected.
- Data sent to an RPAU through a chain of RPAUs.
- No additional computation overhead



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- FPGA Constraints
 - > The FPGA is split into four SLRs.
 - Connected by a limited number of wires.
- Some operations require exchanging the residue polynomials between RPAUs
- Placement of 10 RPAUs using "ring" interconnect





Floorplan of the design




Full system overview



Figure 8: CPU-FPGA interface and software stack

FPGA is used as an accelerator card of a server. HW/SW codesign is used to run applications.

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FPGA Acceleration results



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Our Group's research: Open Problems in FHE

- 1. How to make hardware accelerators for larger parameter sets?
- 2. How to support different parameters?
- 3. How to support different FHE schemes?
- 4. How to implement FHE Bootstrapping?
- 5. From FPGA to ASIC accelerators
 - More parallel processing
 - Custom memory
 - Higher clock frequency and lower power consumption