Questionnaire “Logic and Computability”

Summer Term 2022

Contents

3 Combinational Equivalence Checking

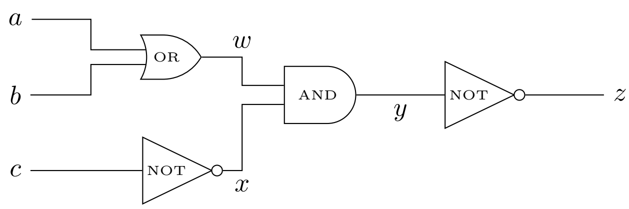
3.1 Lecture

3.1.1 Translating a Circuit into a Formula

1. [Lecture] Explain the algorithm of how to decide the equivalence of combinational circuits via the reduction to satisfiability.

2. [Lecture] Explain the process of translating a combinational circuit into a propositional formula. Draw a combinational circuit with 2 or 3 gates and give the corresponding propositional formula.

3. [Lecture] Compute the propositional formula of the following circuit.



3.1.2 Relations between Satisfiability, Validity, Equivalence and Semantic Entailment

4. [Lecture] Explain the duality of *satisfiability* and *validity* and additional provide examples that show the duality.

5. [Lecture] How can you check whether it is true that $\varphi \models \psi$ using a decision procedure for (a) *satisfiability* or (b) *validity*?

3.1.3 Normal Forms

6. [Lecture] Explain the following terms and give examples: (a) literal, (b) cube, and (c) clause.

7. [Lecture] Given the formula $\varphi = (q \rightarrow p) \wedge (r \vee \neg p)$. Compute its representation in Disjunctive Normal Form (*DNF*) using a truth table.

8. [Lecture] Given the formula $\varphi = (q \rightarrow p) \wedge (r \vee \neg p)$. Compute its representation in Conjunctive Normal Form (*CNF*) using a truth table.

3.1.4 Tseitin Encoding

We list the *Tseitin-rewriting rules* to be applied for the following examples.

$$\chi \leftrightarrow (\varphi \vee \psi) \quad \Leftrightarrow \quad (\neg\varphi \vee \chi) \wedge (\neg\psi \vee \chi) \wedge (\neg\chi \vee \varphi \vee \psi) \tag{1}$$
$$\chi \leftrightarrow (\varphi \wedge \psi) \quad \Leftrightarrow \quad (\neg\chi \vee \varphi) \wedge (\neg\chi \vee \psi) \wedge (\neg\varphi \vee \neg\psi \vee \chi) \tag{2}$$
$$\chi \leftrightarrow \neg\varphi \quad \Leftrightarrow \quad (\neg\chi \vee \neg\varphi) \wedge (\varphi \vee \chi) \tag{3}$$

9. [Lecture] What is the advantage of applying *Tseitin encoding* to obtain a CNF, especially compared to using truth tables?

10. [Lecture] Derive a Rewrite-Rule for an implication node, i.e., what clauses are introduced by the node $x \leftrightarrow (p \rightarrow q)$?

11. [Lecture] Explain the concept of equisatisfiability. Given a propositional logic formula φ, the Tseitin algorithm computes an equisatisfiable formula $CNF(\varphi)$ in CNF. Why is this enough for equivalence checking?

12. [Lecture] Apply Tseitin's encoding to the following formula: $\varphi = \neg(a \vee \neg b) \vee (\neg a \wedge c)$. For each variable you introduce, clearly indicate which subformula of φ it represents.

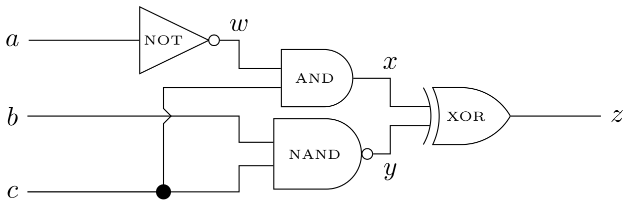
3.1.5 CEC Example

13. [Lecture] Check whether $\varphi_1 = a \wedge \neg b$ and $\varphi_2 = \neg(\neg a \vee b)$ are semantically equivalent using the reduction to satisfiability. Prepare everything until you have a formula CNF(φ), that you can give to a SAT solver.

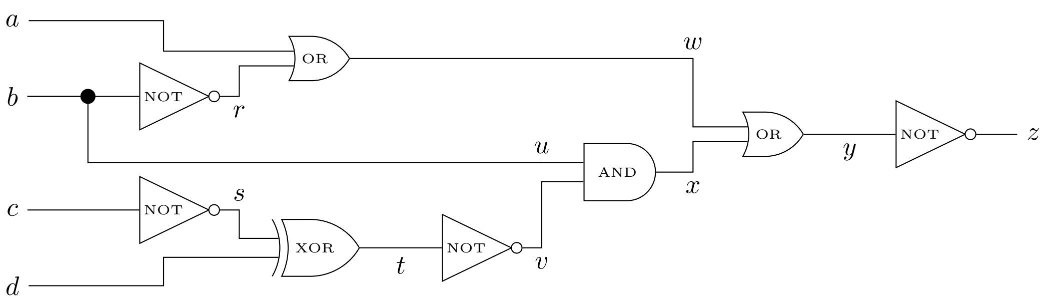
3.2 Self-Assessment

3.2.1 Translating a Circuit into a Formula

14. [Self-Assessment] Compute the propositional formula of the following circuit.



15. [Self-Assessment] Compute the propositional formula of the following circuit.



3.2.2 Relations between Satisfiability, Validity, Equivalence and Entailment

16. [Self-Assessment] *A formula φ is valid, if and only if $\neg\varphi$ is not satisfiable.* Explain why this statement holds in your own words.

17. [Self-Assessment] Given two propositional logic formulas φ and ψ. How can we check whether $\varphi \equiv \psi$ using a decision procedure for (a) satisfiability, (b) for validity, and (c) for semantic entailment?

18. [Self-Assessment] Given a propositional logic formula φ. How can we check whether φ is *valid* using a decision procedure for (a) satisfiability and (b) equivalence?

19. [Self-Assessment] Given a propositional logic formula φ. Tick all statements that are true.

    - ☐ A formula φ is *valid*, if and only if $\neg\varphi$ is *satisfiable*.
    - ☐ A formula ψ is *satisfiable*, if and only if $\neg\varphi$ is *valid*.

- ☐ A formula φ is *satisfiable*, if and only if $\neg\varphi$ is *not valid*.
- ☐ A formula φ is *valid*, if and only if $\neg\varphi$ is *not satisfiable*.

20. [Self-Assessment] Given two propositional logic formulas φ and ψ. Tick all statements that are true.
    - ☐ If $\neg\varphi$ is not satisfiable, φ is not valid.
    - ☐ If $\top \models \varphi$, φ is valid.
    - ☐ If $\varphi \leftrightarrow \psi$ is valid, φ entails ψ.
    - ☐ If $\varphi \rightarrow \psi$ is valid, both formulas are equivalent.
21. [Self-Assessment] Given two propositional logic formulas φ and ψ. Tick all statements that are true.
    - ☐ If $\varphi \wedge \neg\psi$ is not satisfiable, φ entails ψ.
    - ☐ If $\neg\varphi$ is not valid, φ is satisfiable.
    - ☐ If φ entails ψ and ψ entails φ, both formulas are equivalent.
    - ☐ If φ is equivalent to $\top$, φ is valid..

3.2.3 Normal Forms

22. [Self-Assessment] Define the *Disjunctive Normal Form (DNF)* of formulas in propositional logic. Use the proper terminology and give an example.
23. [Self-Assessment] Define the *Conjunctive Normal Form (CNF)* of formulas in propositional logic. Use the proper terminology and give an example.
24. [Self-Assessment] Tick all statements that are true.
    - ☐ A *clause* is a disjunction of literals.
    - ☐ A *clause* is a conjunction of literals.
    - ☐ A *cube* is disjunction of literals.
    - ☐ A *cube* is a conjunction of literals.
25. [Self-Assessment] Given the formula φ with the variables $x_1, ..., x_n$. Tick all statements that are true.
    - ☐ A *literal* is a variables x_i or its negation.
    - ☐ A *literal* forms a formula in conjunctive normal form.
    - ☐ A *literal* forms a formula in disjunctive normal form.
    - ☐ A *literal* is called *positive*, if it is the negation of a variable.
    - ☐ A *literal* is called *negative*, if it is the negation of a variable.
26. [Self-Assessment] Look a the following statements and tick all items that conform to a *DNF*.
    - ☐ $a \vee b$
    - ☐ A DNF is a conjunction of clauses.
    - ☐ $(a \vee b) \wedge (\neg b \vee \neg a \vee c) \wedge \neg c$
    - ☐ $(a \wedge b) \vee (\neg b \wedge \neg a \wedge c) \vee \neg c$
    - ☐ A DNF is a conjunction of disjunctions of literals.
    - ☐ b

- ☐ $a \wedge b \wedge \neg c$
- ☐ $(\neg a \wedge b) \wedge (\neg a \wedge c)$
- ☐ A DNF is a disjunction of cubes.
- ☐ $\neg(a \wedge \neg b) \wedge c$
- ☐ A DNF is a disjunction of conjunctions of literals.
- ☐ $a \wedge \neg b$

27. [Self-Assessment] Tick each correct ending of the following sentence. "A *Conjunctive Normal Form* is ...

- ☐ ...a conjunction of disjunctions of literals."
- ☐ ...a conjunction of clauses."
- ☐ ...a formula that consists only of logical AND operations on sub-formulas which only consist of OR operations on just variables and negations of variables."

28. [Self-Assessment] SAT solvers usually require input formulas to be in *Conjunctive Normal Form* (CNF). In the following list, tick all items that conform to CNF.

- ☐ A formula φ that consists of a conjunction of clauses $c_1, c_2, \ldots, c_n$.
- ☐ A formula φ that consists of a disjunction of clauses $c_1, c_2, \ldots, c_n$.
- ☐ A formula φ that consists of a conjunction of cubes $c_1, c_2, \ldots, c_n$.
- ☐ A formula φ that consists of a disjunction of cubes $c_1, c_2, \ldots, c_n$.
- ☐ A literal l.

29. [Self-Assessment] In the following list, tick all items that conform to the Conjunctive Normal Form (CNF).

- ☐ $(a \wedge b \wedge \neg c) \vee (\neg b \wedge \neg c) \vee (e \wedge \neg f)$
- ☐ a
- ☐ $\neg b$
- ☐ $a \wedge \neg b$
- ☐ $a \vee \neg b$
- ☐ $a \vee (\neg b \wedge c)$
- ☐ $(a \vee \neg b) \wedge c$
- ☐ $\neg(p \vee q)$
- ☐ $x \vee \neg y \vee z$

30. [Self-Assessment] In the following list, tick all items that conform to the Disjunctive Normal Form (DNF).

- ☐ $(a \wedge b \wedge \neg c) \vee (\neg b \wedge \neg c) \vee (e \wedge \neg f)$
- ☐ $(a \vee b \vee \neg c) \wedge (\neg b \vee \neg c) \wedge (e \vee \neg f)$
- ☐ $\neg b$
- ☐ $a \wedge \neg b$
- ☐ $a \vee \neg b$
- ☐ $a \vee (\neg b \wedge c)$
- ☐ $(a \vee \neg b) \wedge c$
- ☐ $\neg(p \vee q)$

☐ $x \vee \neg y \vee z$

31. [Self-Assessment] Given a formula in propositional logic. Explain how to extract a *CNF* representation as well as a *DNF* representation of φ using the truth table from φ.

32. [Self-Assessment] Given the formula $\varphi = (a \wedge \neg b \wedge \neg c) \vee ((\neg c \to a) \to b)$. Use the truth table of φ to compute its representation in (a) CNF and (b) DNF.

33. [Self-Assessment] Given the formula $\varphi = (q \to \neg r) \wedge \neg(p \vee q \vee \neg r)$. Use the truth table of φ to compute its representation in (a) CNF and (b) DNF.

34. [Self-Assessment] Consider the propositional formula $\varphi = (p \vee \neg q) \to (\neg p \wedge \neg r)$. Fill out the truth table for φ and its subformulas. Compute a CNF as well as a DNF for φ from the truth table.

p	q	r	$\neg q$	$p \vee \neg q$	$\neg p$	$\neg r$	$\neg p \wedge \neg r$	$\varphi = (p \vee \neg q) \to (\neg p \wedge \neg r)$
F	**F**	**F**						
F	**F**	**T**						
F	**T**	**F**						
F	**T**	**T**						
T	**F**	**F**						
T	**F**	**T**						
T	**T**	**F**						
T	**T**	**T**						

35. [Self-Assessment] Given the formula $\varphi = \neg(a \to \neg b) \vee (\neg a \to c)$. Use the truth table of φ to compute its representation in (a) CNF and (b) DNF.

36. [Self-Assessment] Consider the propositional formula $\varphi = (\neg(\neg a \wedge b) \wedge \neg c)$. Fill out the truth table for φ and its subformulas. Compute a CNF as well as a DNF for φ from the truth table.

a	b	c	$\neg a$	$\neg a \wedge b$	$\neg(\neg a \wedge b)$	$\neg c$	$\varphi = (\neg(\neg a \wedge b) \wedge \neg c)$
F	**F**	**F**					
F	**F**	**T**					
F	**T**	**F**					
F	**T**	**T**					
T	**F**	**F**					
T	**F**	**T**					
T	**T**	**F**					
T	**T**	**T**					

3.2.4 Tseitin Encoding

Consider the following logic equivalences when applying Tseitin's encoding:

$$\chi \leftrightarrow (\varphi \vee \psi) \quad \Leftrightarrow \quad (\neg\varphi \vee \chi) \wedge (\neg\psi \vee \chi) \wedge (\neg\chi \vee \varphi \vee \psi) \tag{4}$$

$$\chi \leftrightarrow (\varphi \wedge \psi) \quad \Leftrightarrow \quad (\neg\chi \vee \varphi) \wedge (\neg\chi \vee \psi) \wedge (\neg\varphi \vee \neg\psi \vee \chi) \tag{5}$$

$$\chi \leftrightarrow \neg\varphi \quad \Leftrightarrow \quad (\neg\chi \vee \neg\varphi) \wedge (\varphi \vee \chi) \tag{6}$$

37. [Self-Assessment] (a) What does it mean that two formulas φ and ψ are *equisatisfiable*? (b) Explain the difference between *satisfiability* and *equisatisfiability*.

38. [Self-Assessment] Suppose you have a propositional formula φ. Let ψ be the result of applying Tseitin's encoding to φ. Is φ *equivalent* to ψ? Provide a reason for your answer and explain the relation between φ and ψ.

39. [Self-Assessment] Explain the concept of *Tseitin's Encoding* to obtain formulas in CNF. Give step-by-step instructions of how to apply Tseitin's encoding to a propositional formula. (Note: Focus on the concept. You do *not* need to quote the rewrite rules!)

40. [Self-Assessment] Derive a Rewrite-Rule for a NAND node, i.e., what clauses are introduced by the node $x \leftrightarrow (p \text{ NAND } q)$?

41. [Self-Assessment] Derive a Rewrite-Rule for a NOR node, i.e., what clauses are introduced by the node $x \leftrightarrow (p \text{ NOR } q)$?

42. [Self-Assessment] Derive a Rewrite-Rule for a XOR node, i.e., what clauses are introduced by the node $x \leftrightarrow (p \oplus q)$?

43. [Self-Assessment] Apply Tseitin's encoding to the following formula:

$$\varphi = \neg(\neg b \wedge \neg c) \vee (\neg c \wedge a).$$

For each variable you introduce, clearly indicate which subformula of φ it represents.

44. [Self-Assessment] Apply Tseitin's encoding to the following formula:

$$\varphi = (q \wedge \neg r) \vee \neg(q \wedge \neg r)$$

. For each variable you introduce, clearly indicate which subformula of φ it represents.

45. [Self-Assessment] Apply Tseitin's encoding to the following formula:

$$\varphi = (\neg(\neg a \wedge b) \wedge \neg c).$$

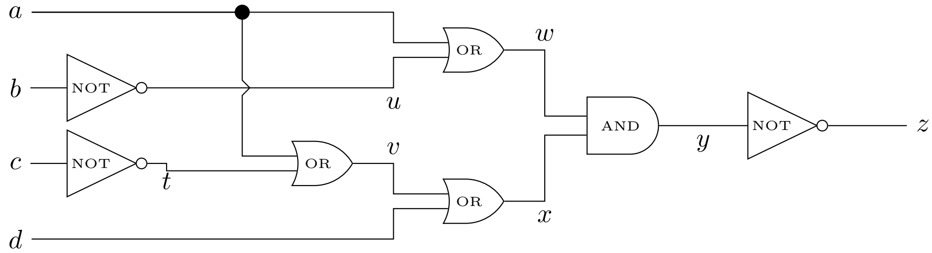
For each variable you introduce, clearly indicate which subformula of φ it represents.

46. [Self-Assessment] Apply Tseitin's encoding to the following formula:

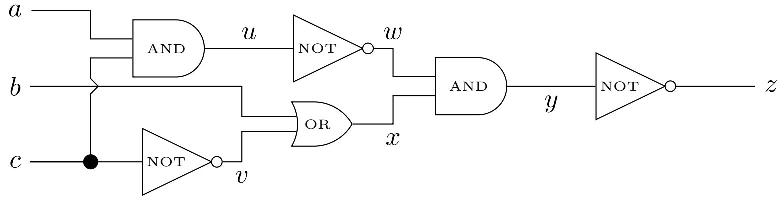
$$\varphi = (p \vee \neg q) \vee (\neg p \wedge \neg r).$$

For each variable you introduce, clearly indicate which subformula of φ it represents.

47. [Self-Assessment] Compute the propositional formula of the following circuit and transform it into an equisatisfiable formula in CNF by applying Tseitin's encoding. For each variable you introduce, clearly indicate which subformula of φ it represents.



48. [Self-Assessment] Compute the propositional formula of the following circuit and transform it into an equisatisfiable formula in CNF by applying Tseitin's encoding. For each variable you introduce, clearly indicate which subformula of φ it represents.



3.2.5 CEC Example

49. [Self-Assessment] Check whether $\varphi_1 = (a \wedge b) \vee \neg c$ and $\varphi_2 = (a \vee \neg c) \wedge (b \vee \neg c)$ are semantically equivalent using the reduction to satisfiability. Prepare everything until you have a formula CNF(φ), that you can give to a SAT solver.