

Questionnaire “Logic and Computability”

Summer Term 2022

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Solution:

Step	0	1	2	3	4	5	ω		
a	0	1	0	1	1	1	1	1	1
b	0	0	0	1	0	1	0	0	0
c	0	0	0	1	0	0	0	1	1
Ga	0	1	0	1	1	1	1	1	1
Fb	0	1	0	1	1	1	0	0	0
$Fb \vee c$	0	1	0	1	1	1	0	1	1
$Ga \rightarrow (Fb \vee c)$	0	1	0	1	1	1	0	1	1

4. [Lecture] Given the following execution word w of a Kripke structure. Evaluate the formula φ on w . Evaluate each sub-formula for any execution step using the provided table.

- $w = \{\}, \{a\}, \{\}, \{a, b, c\}, \{a\}, \{a, b\}, (\{a\}, \{a, c\}, \{a, c\})^\omega$
- $\varphi = GFa \rightarrow (FG\neg b \wedge c)$

Step	0	1	2	3	4	5	ω		
a	0	1	0	1	1	1	1	1	1
b	0	0	0	1	0	1	0	0	0
c	0	0	0	1	0	0	0	1	1
GFa									
$FG\neg b$									
$FG\neg b \wedge c$									
$GFa \rightarrow (FG\neg b \wedge c)$									

Solution:

Step	0	1	2	3	4	5	ω		
a	0	1	0	1	1	1	1	1	1
b	0	0	0	1	0	1	0	0	0
c	0	0	0	1	0	0	0	1	1
GFa	1	1	1	1	1	1	1	1	1
$FG\neg b$	1	1	1	1	1	1	1	1	1
$FG\neg b \wedge c$	0	0	0	1	0	0	0	1	1
$GFa \rightarrow (FG\neg b \wedge c)$	0	0	0	1	0	0	0	1	1

5. [Lecture] Translate the following sentences in computation tree logic CTL^* .

- For any execution, it always holds that whenever the robot visits region A, it visits region C within the next two steps.
- There exists an execution such that the robot visits region C within the next two steps after visiting region A.

Solution:

We use the following Boolean variables:

- a represents “The robot visits region A”
- b represents “The robot visits region B”
- c represents “The robot visits region C”
- $\varphi_1 := AG(a \rightarrow Xc \vee XXc)$
- $\varphi_2 := EG(a \rightarrow Xc \vee XXc)$

6. [Lecture] Translate the following sentences in computation tree logic CTL^* .

- The robot can visit region A infinitely often and region C infinitely often
- Always, the robot visits region A infinitely often and region C infinitely often.
- If the robot visits region A infinitely often, it should also visit region C finitely often.

Solution:

We use the following Boolean variables:

- a represents “The robot visits region A”
- b represents “The robot visits region B”
- c represents “The robot visits region C”
- $\varphi_1 := E(GFa \wedge GFc)$
- $\varphi_2 := A(GFa \wedge FG\neg c)$
- $\varphi_3 := A(GFa \rightarrow GFc)$

7. [Lecture] Given the following Kripke structure \mathcal{K} . Does the initial state s_0 of \mathcal{K} satisfy the following formulas?

- $\varphi_1 := EXX(a \wedge b)$
- $\varphi_2 := EXAX(a \wedge b)$

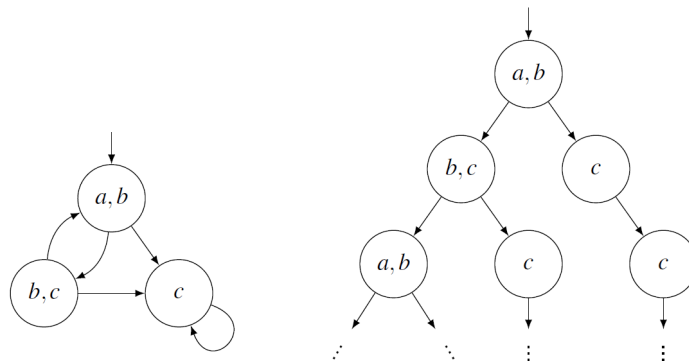


Figure 1: Left: Kripke structure of Example 7, Right: Corresponding computation tree

Solution:

- $s_0 \models EXX(a \wedge b)$
- $s_0 \not\models EXAX(a \wedge b)$

8. [Lecture] Given the following Kripke structure \mathcal{K} . Does the initial state s_0 of \mathcal{K} satisfy the following formulas?

- $\varphi_1 := EXp$
- $\varphi_2 := EG\neg p$

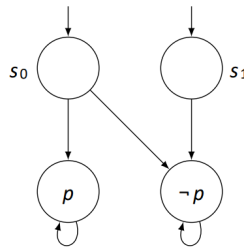


Figure 2: Kripke structure of Example 8

Solution:

- $s_0 \models EXp$
- $s_0 \models EG\neg p$

10.2 Self-Assessment

9. [Self-Assessment] Give the definition of a *Kripke structure*. Explain the components of the tuple a Kripke structure consists of. Give an example of a Kripke structure in the representation of a graph.

Solution:

10. [Self-Assessment] Give the definition of *paths* and *words* of Kripke structures. Give an example in which you draw a graph representing a Kripke structure, and give one possible infinite path and corresponding word.

Solution:

11. [Self-Assessment] What does a *computation tree* of a Kripke structure represent? Give an example in which you draw a graph representing a Kripke structure, and draw the first 3 levels of the computation tree of this Kripke structure.

Solution:

12. [Self-Assessment]

The temporal operators describe properties that hold along a given infinite path ρ through the computation tree of a Kripke structure. Given two formulas φ and ψ describing state properties.

- Which are the properties that ρ needs to satisfy such that $\rho \models G\varphi$?
- Which are the properties that ρ needs to satisfy such that $\rho \models F\varphi$?
- Which are the properties that ρ needs to satisfy such that $\rho \models X\varphi$?
- Which are the properties that ρ needs to satisfy such that $\rho \models \varphi U \psi$?

Solution:

13. [Self-Assessment] Consider an ordinary traffic junction with incoming lanes from the north, south, east and west. We want to formulate relevant constraints that a traffic light system has to fulfill.

Give a set of propositional variables that model whether the north and south *or* the east and the west get the

- green,
- yellow or
- red

light, respectively.

Formulate the following sentences using CTL^* :

- (a) The north/south lanes will never get the green light at same time as the east/west lanes.
- (b) Whenever the north/south lane receive the green light it will stay green until it changes to yellow.
- (c) When the east/west lane has the red light, it will eventually get the yellow and red light until the light switches to green.

Solution:

14. [Self-Assessment] Given the following execution word w of a Kripke structure. Evaluate the formula φ on w . Evaluate each sub-formula for any execution step using the provided table.

- $w = \{\}, \{a\}, \{\}, \{a, b\}, \{a\}, \{a, b\}, (\{a\}, \{a, b\}, \{a\})^\omega$
- $\varphi = FGa \rightarrow FGb$

Step	0	1	2	3	4	5	ω		
a	0	1	0	1	1	1	1	1	1
b	0	0	0	1	0	1	0	1	0
FGa									
FGb									
$FGa \rightarrow FGb$									

Solution:

15. [Self-Assessment] Given the following execution word w of a Kripke structure. Evaluate the formula φ on w . Evaluate each sub-formula for any execution step using the provided table.

- $w = \{a\}, \{a\}, \{a\}, \{b, c\}, \{a\}, \{a, b\}(\{a\}, \{c\})^\omega$
- $\varphi = aUc \vee Fb$

Step	0	1	2	3	4	5	ω	
a	1	1	1	0	1	1	1	0
b	0	0	0	1	0	1	0	0
c	0	0	0	1	0	0	0	1
aUc								
Fb								
$aUc \vee Fb$								

Solution:

16. [Self-Assessment] Give the definition of the syntax of the computation tree logic CTL^* . In particular, give the definition of state formulas and path formulas.

Solution:

17. [Self-Assessment] Give an intuitive explanation of the semantics of computation tree logic CTL^* . Therefore, explain the semantics of the introduced path quantifiers and temporal operators with respect to the computation tree of a Kripke structure.

Solution: