Logic and Computability SS22
Assignment 4

Practical Session: June 02, 2022

1. [Practicals] **[3 Points]** Given the formula:

   $\varphi_{EUF} := f(x) = y \wedge x = g(x) \vee x \neq f(x) \wedge g(x) = f(g(x)) \vee y \neq g(x) \wedge x = f(y) \wedge g(y) = f(g(x))$

   Apply the *Ackermann* reduction algorithm to compute an equisatisfiable formula in $\mathcal{T}_E$.

2. [Practicals] **[3 Points]** Given the formula:

   $\varphi_{EUF} \quad := \quad x = f(x,y) \wedge x \neq y \leftrightarrow z = f(x,y) \vee f(y,z) \neq z \wedge y \neq f(x,y) \vee y = f(x,z)$

   Apply the *Ackermann* reduction algorithm to compute an equisatisfiable formula in $\mathcal{T}_E$.

3. [Practicals] **[3 Points]** Perform graph-based reduction to translate the following formula in $\mathcal{T}_E$ into an equisatisfiable formula in propositional logic.

   $$\varphi_E := x \neq y \wedge y = c \vee c = d \rightarrow \neg(d \neq z \vee z = a) \wedge \neg(a = b \wedge x \neq z).$$

4. [Practicals] **[5 Points]** Consider the following formula in $\mathcal{T}_{EUF}$:

   $$\varphi_{EUF} \quad := (y = z \vee f(x) = f(y)) \rightarrow (x = z \vee f(x) = x \wedge f(x) = y)$$

   Use Ackermann's reduction to compute an equisatisfiable formula in $\mathcal{T}_E$.

   Then perform the graph-based reduction on the outcome of Ackermann's reduction to construct an equisatisfiable propositional formula.

5. [Practicals] **[3 Points]** Use the Congruence-Closure algorithm to check if the following assignment for the equalities is satisfiable.

   $\varphi_{EUF} \quad := \quad f(b) = a \wedge e = b \wedge c = f(c) \wedge d \neq f(e) \wedge f(a) = f(d) \wedge a \neq f(c) \wedge d = f(a)$

6. [Practicals] **[3 Points]** Use the Congruence-Closure algorithm to check if the following assignment for the equalities is satisfiable.

   $$\begin{aligned}\varphi_{EUF} \quad := \quad & f(o) = k \wedge l \neq f(m) \wedge n \neq l \wedge f(k) = m \wedge f(o) = f(k) \wedge o \neq k \wedge \\ & l \neq f(n) \wedge f(m) \neq k \wedge m \neq f(m) \wedge o = n \wedge f(m) = o\end{aligned}$$