

Logic and Computability SS22

Assignment 4

Practical Session: June 06, 2022

For each of the following sequents, either provide a natural deduction proof, or a counterexample that proves the sequent invalid.

For proofs, clearly indicate which rule, and what assumptions/premises/intermediate results you are using in each step. Also clearly indicate the scope of any boxes you use.

For counterexamples, give a complete model. Show that the model satisfies the premise(s) of the sequent in question, but does not satisfy the respective conclusion.

1. [Practicals] [2 Point]

$$(a) (\forall x (\neg A(x))) \vee (\exists x (B(x))) \quad \vdash \quad \forall x (\neg A(x) \vee B(x))$$

$$(b) (\forall x (\neg A(x))) \vee (\exists x (B(x))) \quad \vdash \quad \exists x (\neg A(x) \vee B(x))$$

2. [Practicals] [2 Point] $\exists x P(x) \vee \exists x Q(x) \quad \vdash \quad \exists x (P(x) \vee Q(x))$

3. [Practicals] [3 Point] $\exists b (a \rightarrow B(b)) \quad \vdash \quad a \rightarrow \exists b B(b)$

4. [Practicals] [4 Point] $\exists x (S(x) \rightarrow T(x)), \neg T(z) \wedge \neg T(y) \quad \vdash \quad \neg S(y)$

5. [Practicals] [4 Point] $\forall r U(r) \wedge \forall r (S(r) \rightarrow T(r)) \quad \vdash \quad \exists r \neg T(r) \rightarrow \exists r (\neg S(r) \wedge U(r))$

6. [Practicals] [5 Point] $\exists a (P(a) \vee Q(a)), \quad \exists a P(a) \rightarrow R(c), \quad \exists b Q(b) \rightarrow R(c) \quad \vdash \quad R(c)$

7. [Practicals] [5 Point][BONUS] $\exists x P(x) \rightarrow \exists x Q(x) \quad \vdash \quad \exists x (P(x) \rightarrow Q(x))$