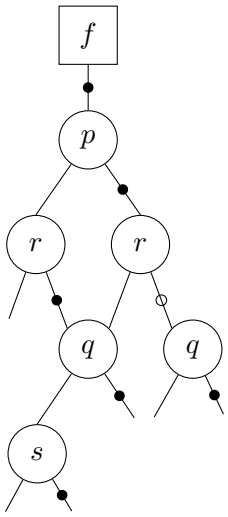
Logic and Computability SS22
Assignment 3

Practical Session: May 13, 2022

1. [Practicals] **[2 Points]**



   (a) Use the BDD shown in the figure on the right to check if the formula it represents evaluates to `true` or `false` with the following variable assignments.

      i. $\mathcal{M}_1 : \; p = \top, r = \bot, q = \top, s = \bot$

      ii. $\mathcal{M}_2 : \; p = \bot, r = \bot, q = \bot, s = \top$

   (b) Find the formula f that is represented by the BDD.

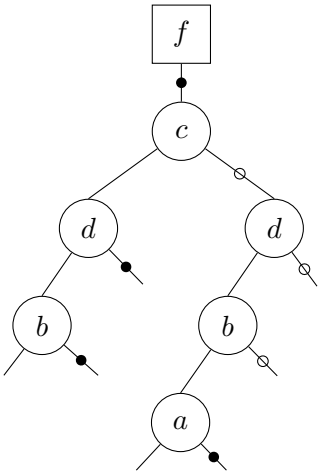
Solution:

(a) i. false

ii. false

(b) $f = (p \wedge \neg r \wedge q \wedge s) \vee (\neg p \wedge r \wedge q \wedge s) \vee (\neg p \wedge \neg r \wedge q)$

2. [Practicals] **[2 Points]**



   (a) Use the BDD shown in the figure on the right to check if the formula it represents evaluates to `true` or `false` with the following variable assignments.

      i. $\mathcal{M}_1 : \; a = \bot, b = \top, c = \bot, d = \top$

      ii. $\mathcal{M}_2 : \; a = \top, b = \top, c = \top, d = \top$

   (b) Find the formula f that is represented by the BDD.

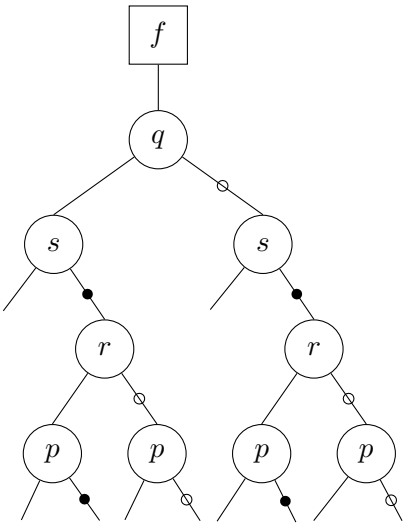
Solution:

(a) i. true

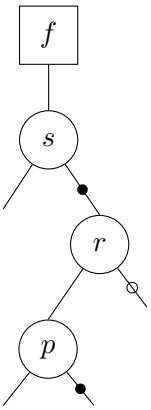
ii. false

(b) $f = (c \wedge \neg d) \vee (c \wedge d \wedge \neg b) \vee (\neg c \wedge d \wedge b \wedge \neg a)$

3. [Practicals] **[2 Points]** Convert the following BDD into a *reduced ordered* BDD.



Solution:



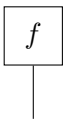
4. [Practicals] **[3 Points]** Construct a ROBDD for the formula

$$f = (a \wedge d \wedge c) \vee (b \wedge \neg d \wedge \neg a) \vee (c \rightarrow \neg d) \vee (a \rightarrow \neg b)$$

using *variable order* $b < a < d < c$. Use complemented edges and a node for `true` as the only constant node. To simplify drawing, you may assume that *dangling edges* point to the constant node. Write down all cofactors that you compute to obtain the final result and mark them in the graph.

Solution:

$f = (a \wedge d \wedge c) \vee (b \wedge \neg d \wedge \neg a) \vee (c \to \neg d) \vee (a \to \neg b)$
$f_b = \top$
$f_{\neg b} = \top$



5. [Practicals] **[3.5 Points]** Construct a reduced ordered binary decision diagram (ROBDD) for the formula

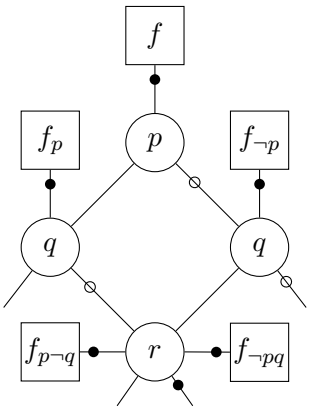
$$f = (p \oplus q) \wedge \neg r$$

using *variable order* $p < q < r$. Use complemented edges and a node for `true` as the only constant node. To simplify drawing, you may assume that *dangling edges* point to the constant node. Write down all cofactors that you compute to obtain the final result and mark them in the graph.

Solution:

The final ROBDD:

$f = (p \oplus q) \wedge \neg r$
$f_p = \neg q \wedge \neg r$
$f_{pq} = \bot$
$f_{p\neg q} = \neg r$
$f_{p\neg qr} = \bot$
$f_{p\neg q\neg r} = \top$
$f_{\neg p} = q \wedge \neg r$
$f_{\neg pq} = \neg r = f_{p\neg q}$
$f_{\neg p\neg q} = \bot$



6. [Practicals] **[3.5 Points]** Construct a ROBDD for the formula

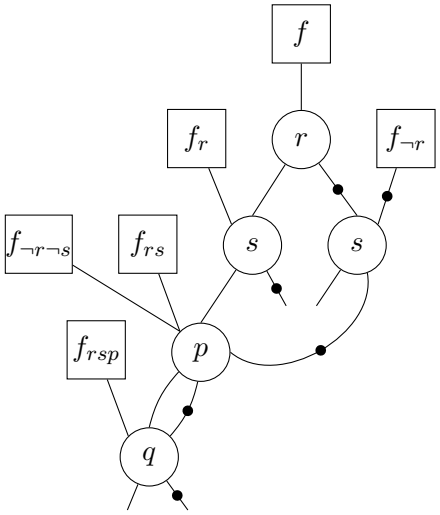
$$f = (p \leftrightarrow q) \wedge (r \leftrightarrow s)$$

using *variable order* $r < s < p < q$. Use complemented edges and a node for `true` as the only constant node. To simplify drawing, you may assume that *dangling edges* point to the constant node. Write down all cofactors that you compute to obtain the final result and mark them in the graph.

Solution:

The final ROBDD:

$f = (p \leftrightarrow q) \wedge (r \leftrightarrow s)$
- $f_r = (p \leftrightarrow q) \wedge s$
  - $f_{rs} = (p \leftrightarrow q)$
    - $f_{rsp} = q$
      - $f_{rspq} = \top$
      - $f_{rsp\neg q} = \bot$
    - $f_{rs\neg p} = \neg q = \neg f_{rsp}$
  - $f_{r\neg s} = \bot$
- $f_{\neg r} = (p \leftrightarrow q) \wedge \neg s$
  - $f_{\neg rs} = \bot$
  - $f_{\neg r\neg s} = (p \leftrightarrow q) = f_{rs}$



7. [Practicals] **[4 Points]**

   (a) Construct a Reduced Ordered Binary Decision Diagram (ROBDD) for the formula

   $$f = (a \vee b \vee c) \wedge \neg d$$

   using *variable order* $c < a < d < b$. Use complemented edges and a node for `true` as the only constant node. To simplify drawing, you may assume that *dangling edges* point to the constant node. Write down all cofactors that you compute to obtain the final result and mark them in the graph.

   (b) Construct a Reduced Ordered Binary Decision Diagram (ROBDD) for f with a different variable order. The ROBDD should result in a *smaller* ROBDD, w.r.t. the number of nodes.

Solution:

8. [Practicals] **[4 Points]**

   (a) Construct a Reduced Ordered Binary Decision Diagram (ROBDD) for the formula

   $$f = (a \vee b \vee c) \wedge \neg d$$

   using *variable order* $c < a < d < b$. Use complemented edges and a node for `true` as the only constant node. To simplify drawing, you may assume that *dangling edges* point to the constant node. Write down all cofactors that you compute to obtain the final result and mark them in the graph.

   (b) Construct a Reduced Ordered Binary Decision Diagram (ROBDD) for f with a different variable order. The ROBDD should result in a *smaller* ROBDD, w.r.t. the number of nodes.