Logic and Computability SS22
Assignment 3

Practical Session: May 13, 2022

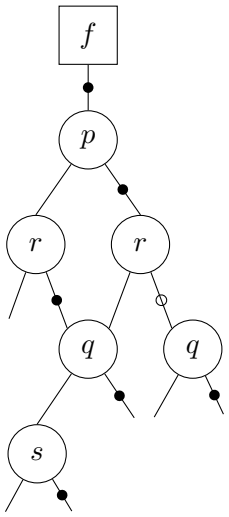
1. [Practicals] **[2 Points]**

   (a) Use the BDD shown in the figure on the right to check if the formula it represents evaluates to `true` or `false` with the following variable assignments.

      i. $\mathcal{M}_1 : \; p = \top, r = \bot, q = \top, s = \bot$

      ii. $\mathcal{M}_2 : \; p = \bot, r = \bot, q = \bot, s = \top$

   (b) Find the formula f that is represented by the BDD.



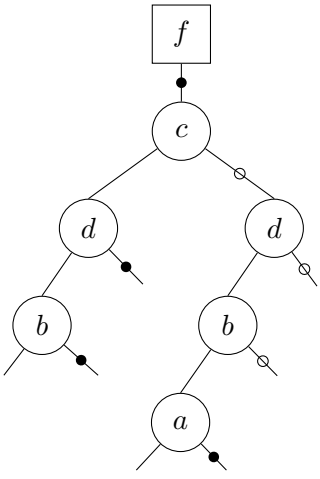
2. [Practicals] **[2 Points]**

   (a) Use the BDD shown in the figure on the right to check if the formula it represents evaluates to `true` or `false` with the following variable assignments.

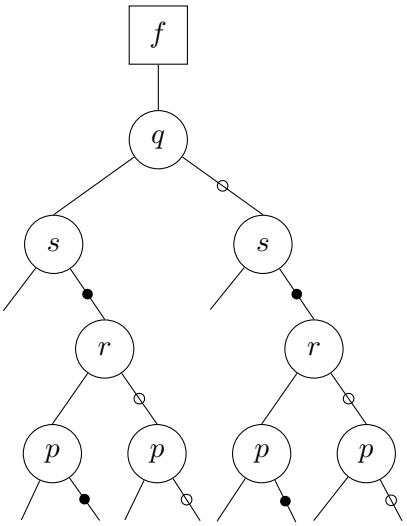
      i. $\mathcal{M}_1 : \; a = \bot, b = \top, c = \bot, d = \top$

      ii. $\mathcal{M}_2 : \; a = \top, b = \top, c = \top, d = \top$

   (b) Find the formula f that is represented by the BDD.



3. [Practicals] **[2 Points]** Convert the following BDD into a *reduced ordered* BDD.



4. [Practicals] **[3 Points]** Construct a ROBDD for the formula

$$f = (a \wedge d \wedge c) \vee (b \wedge \neg d \wedge \neg a) \vee (c \rightarrow \neg d) \vee (a \rightarrow \neg b)$$

using *variable order* $b < a < d < c$. Use complemented edges and a node for `true` as the only constant node. To simplify drawing, you may assume that *dangling edges* point to the constant node. Write down all cofactors that you compute to obtain the final result and mark them in the graph.

5. [Practicals] **[3.5 Points]** Construct a reduced ordered binary decision diagram (ROBDD) for the formula

$$f = (p \oplus q) \wedge \neg r$$

using *variable order* $p < q < r$. Use complemented edges and a node for `true` as the only constant node. To simplify drawing, you may assume that *dangling edges* point to the constant node. Write down all cofactors that you compute to obtain the final result and mark them in the graph.

6. [Practicals] **[3.5 Points]** Construct a ROBDD for the formula

$$f = (p \leftrightarrow q) \wedge (r \leftrightarrow s)$$

using *variable order* $r < s < p < q$. Use complemented edges and a node for `true` as the only constant node. To simplify drawing, you may assume that *dangling edges* point to the constant node. Write down all cofactors that you compute to obtain the final result and mark them in the graph.

7. [Practicals] **[4 Points]**

(a) Construct a Reduced Ordered Binary Decision Diagram (ROBDD) for the formula

$$f = (a \vee b \vee c) \wedge \neg d$$

using *variable order* $c < a < d < b$. Use complemented edges and a node for `true` as the only constant node. To simplify drawing, you may assume that *dangling edges* point to the constant node. Write down all cofactors that you compute to obtain the final result and mark them in the graph.

(b) Construct a Reduced Ordered Binary Decision Diagram (ROBDD) for f with a different variable order. The ROBDD should result in a *smaller* ROBDD, w.r.t. the number of nodes.