

# Logic and Computability SS22

## Assignment 2

Practical Session: 29. April 2022

For the following exercises use the DPLL algorithm (including Boolean Constraint Propagation (BCP), pure literals, and conflict-driven clause learning) to check on paper, if the following CNF formulas are satisfiable.

If the formula is satisfiable, give a satisfying model, else show a complete resolution proof for the formula's unsatisfiability.

- Write down all the steps of the DPLL algorithm,
- draw the conflict graphs,
- and state the resolution proofs for all learned clauses

### Rules:

- When resolving a conflict, only undo the last decision.
- Choose variables for decisions, BCP and pure literals in alphabetical order, starting with the *negative* phase ( $\neg a > a > \neg b > b \dots$ ).
- Always try to perform BCP first, before checking for pure literals, before making a decision.

### 1. [Practicals] [2 Points]

Clause 1:  $\{a, b, c\}$

Clause 2:  $\{\neg a, \neg b, \neg c\}$

Clause 3:  $\{a, c, \neg e\}$

Clause 4:  $\{\neg b, \neg c, e\}$

Clause 5:  $\{b, e\}$

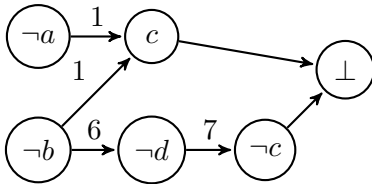
Clause 6:  $\{b, \neg d\}$

Clause 7:  $\{\neg c, d\}$

Clause 8:  $\{\neg c, e\}$

Solution:

Step	1	2	3	4	5
Decision Level	0	1	2	2	2
Assignment	-	$\neg a$	$\neg a, \neg b$	$\neg a, \neg b, c$	$\neg a, \neg b, c, \neg d$
Cl. 1: $a, b, c$	$a, b, c$	$b, c$	$c$	✓	✓
Cl. 2: $\neg a, \neg b, \neg c$	$\neg a, \neg b, \neg c$	✓	✓	✓	✓
Cl. 3: $a, c, \neg e$	$a, c, \neg e$	$c, \neg e$	$c, \neg e$	✓	✓
Cl. 4: $\neg b, \neg c, e$	$\neg b, \neg c, e$	$\neg b, \neg c, e$	✓	✓	✓
Cl. 5: $b, e$	$b, e$	$b, e$	$e$	$e$	$e$
Cl. 6: $b, \neg d$	$b, \neg d$	$b, \neg d$	$\neg d$	$\neg d$	✓
Cl. 7: $\neg c, d$	$\neg c, d$	$\neg c, d$	$\neg c, d$	$d$	$\}$ ✗
Cl. 8: $\neg c, e$	$\neg c, e$	$\neg c, e$	$\neg c, e$	$e$	$e$
BCP	-	-	$c$	$\neg d$	-
PL	-	-	-	-	-
Decision	$\neg a$	$\neg b$	-	-	-



$$\frac{7. \neg c \vee d}{a \vee b \vee d} \quad \frac{1. a \vee b \vee c}{a \vee b} \quad 6. b \vee \neg d$$

Step	(1)	6	7	8	9
Decision Level	1	1	1	2	2
Assignment	$\neg a$	$\neg a, b$	$\neg a, b, d$	$\neg a, b, d, \neg c$	$\neg a, b, d, \neg c, \neg e$
Cl. 1: $a, b, c$	$b, c$	✓	✓	✓	✓
Cl. 2: $\neg a, \neg b, \neg c$	✓	✓	✓	✓	✓
Cl. 3: $a, c, \neg e$	$c, \neg e$	$c, \neg e$	$c, \neg e$	$\neg e$	✓
Cl. 4: $\neg b, \neg c, e$	$\neg b, \neg c, e$	$\neg c, e$	$\neg c, e$	✓	✓
Cl. 5: $b, e$	$b, e$	✓	✓	✓	✓
Cl. 6: $b, \neg d$	$b, \neg d$	✓	✓	✓	✓
Cl. 7: $\neg c, d$	$\neg c, d$	$\neg c, d$	✓	✓	✓
Cl. 8: $\neg c, e$	$\neg c, e$	$\neg c, e$	$\neg c, e$	✓	✓
Cl. 9: $a, b$	$b$	✓	✓	✓	✓
BCP	$b$	-	-	$\neg e$	-
PL	-	$d$	-	-	-
Decision	-	-	$\neg c$	-	SAT

2. [Practicals] [2.5 Points]

Clause 1:  $\{\neg a, c\}$

Clause 2:  $\{\neg a, b, \neg c\}$

Clause 3:  $\{\neg b, e\}$

Clause 4:  $\{a, d\}$

Clause 5:  $\{a, \neg c\}$

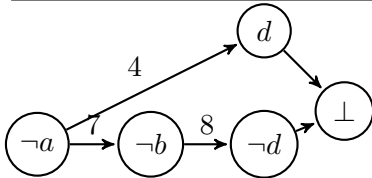
Clause 6:  $\{\neg a, \neg e\}$

Clause 7:  $\{a, \neg b\}$

Clause 8:  $\{b, \neg d\}$

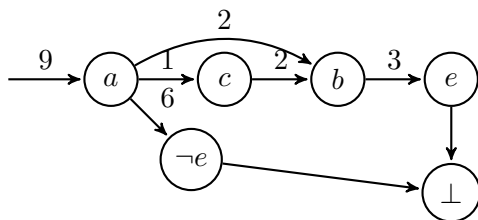
Solution:

Step	1	2	3	4	5
Decision Level	0	1	1	1	1
Assignment	-	$\neg a$	$\neg a, \neg b$	$\neg a, \neg b, \neg c$	$\neg a, \neg b, \neg c, \neg d$
Cl. 1: $\neg a, c$	$\neg a, c$	✓	✓	✓	✓
Cl. 2: $\neg a, b, \neg c$	$\neg a, b, \neg c$	✓	✓	✓	✓
Cl. 3: $\neg b, e$	$\neg b, e$	$\neg b, e$	✓	✓	✓
Cl. 4: $a, d$	$a, d$	$d$	$d$	$d$	{ } ✗
Cl. 5: $a, \neg c$	$a, \neg c$	$\neg c$	$\neg c$	✓	✓
Cl. 6: $\neg a, \neg e$	$\neg a, \neg e$	✓	✓	✓	✓
Cl. 7: $a, \neg b$	$a, \neg b$	$\neg b$	✓	✓	✓
Cl. 8: $b, \neg d$	$b, \neg d$	$b, \neg d$	$\neg d$	$\neg d$	✓
BCP	-	$\neg b$	$\neg c$	$\neg d$	-
PL	-	-	-	-	-
Decision	$\neg a$	-	-	-	-



$$\frac{\frac{8. b \vee \neg d \quad 4. a \vee d}{a \vee b} \quad 7. a \vee \neg b}{a}$$

Step	(1)	6	7	8	9
Decision Level	0	0	0	0	0
Assignment	-	$a$	$a, c$	$a, c, b$	$a, c, b, \neg e$
Cl. 1: $\neg a, c$	$\neg a, c$	$c$	✓	✓	✓
Cl. 2: $\neg a, b, \neg c$	$\neg a, b, \neg c$	$b, \neg c$	$b$	✓	✓
Cl. 3: $\neg b, e$	$\neg b, e$	$\neg b, e$	$\neg b, e$	$e$	{ } ✗
Cl. 4: $a, d$	$a, d$	✓	✓	✓	✓
Cl. 5: $a, \neg c$	$a, \neg c$	✓	✓	✓	✓
Cl. 6: $\neg a, \neg e$	$\neg a, \neg e$	$\neg e$	$\neg e$	$\neg e$	✓
Cl. 7: $a, \neg b$	$a, \neg b$	✓	✓	✓	✓
Cl. 8: $b, \neg d$	$b, \neg d$	$b, \neg d$	$b, \neg d$	✓	✓
Cl. 9: $a$	$a$	✓	✓	✓	✓
BCP	$a$	$c$	$b$	$\neg e$	-
PL	-	-	-	-	-
Decision	-	-	-	-	UNSAT



$$\frac{\frac{\frac{3. \neg b \vee e \quad 6. \neg a \vee \neg e}{\neg a \vee \neg b} \quad 2. \neg a \vee b \vee \neg c}{\neg a \vee \neg a} \quad 1. \neg a \vee c \quad 9. a}{\perp}$$

3. [Practicals] [2.5 Points]

Clause 1:  $\{a, \neg b, c\}$

Clause 2:  $\{b, \neg c, d\}$

Clause 3:  $\{a, \neg b\}$

Clause 4:  $\{a, c\}$

Clause 5:  $\{\neg c, \neg d\}$

Solution:

Step	1	2	3	4
Decision Level	0	0	0	0
Assignment	-	$a$	$a, b, \neg c$	$a, b, \neg c$
Cl. 1: $a, \neg b, c$	$a, \neg b, c$	✓	✓	✓
Cl. 2: $b, \neg c, d$	$b, \neg c, d$	$b, \neg c, d$	✓	✓
Cl. 3: $a, \neg b$	$a, \neg b$	✓	✓	✓
Cl. 4: $a, c$	$a, c$	✓	✓	✓
Cl. 5: $\neg c, \neg d$	$\neg c, \neg d$	$\neg c, \neg d$	$\neg c, \neg d$	✓
BCP	-	-	-	-
PL	$a$	$b$	$\neg c$	-
Decision	-	-	-	SAT

4. [Practicals] [3 Points]

Clause 1:  $\{a, \neg b\}$

Clause 2:  $\{a, c\}$

Clause 3:  $\{\neg a, e\}$

Clause 4:  $\{b, c\}$

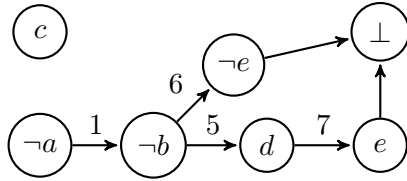
Clause 5:  $\{b, d\}$

Clause 6:  $\{b, \neg e\}$

Clause 7:  $\{\neg d, e\}$

Solution:

Step	1	2	3	4	5	6
Decision Level	0	0	1	1	1	1
Assignment	-	$c$	$c, \neg a$	$c, \neg a, \neg b$	$c, \neg a, \neg b, d$	$c, \neg a, \neg b, d, \neg e$
Cl. 1: $a, \neg b$	$a, \neg b$	$a, \neg b$	$\neg b$	✓	✓	✓
Cl. 2: $a, c$	$a, c$	✓	✓	✓	✓	✓
Cl. 3: $\neg a, e$	$\neg a, e$	$\neg a, e$	✓	✓	✓	✓
Cl. 4: $b, c$	$b, c$	✓	✓	✓	✓	✓
Cl. 5: $b, d$	$b, d$	$b, d$	$b, d$	$d$	✓	✓
Cl. 6: $b, \neg e$	$b, \neg e$	$b, \neg e$	$b, \neg e$	$\neg e$	$\neg e$	✓
Cl. 7: $\neg d, e$	$\neg d, e$	$\neg d, e$	$\neg d, e$	$\neg d, e$	$e$	{ } ✗
BCP	-	-	$\neg b$	$d$	$\neg e$	-
PL	$c$	-	-	-	-	-
Decision	-	$\neg a$	-	-	-	-



$$\frac{\frac{\frac{7. \neg d \vee e \quad 6. b \vee \neg e}{\neg d \vee b} \quad 5. b \vee d}{b} \quad 1. a \vee \neg b}{a}$$

Step	(1)	7	8	9
Decision Level	0	0	0	0
Assignment	-	$a$	$a, e$	$a, e, b$
Cl. 1: $a, \neg b$	$a, \neg b$	✓	✓	✓
Cl. 2: $a, c$	$a, c$	✓	✓	✓
Cl. 3: $\neg a, e$	$\neg a, e$	$e$	✓	✓
Cl. 4: $b, c$	$b, c$	$b, c$	$b, c$	✓
Cl. 5: $b, d$	$b, d$	$b, d$	$b, d$	✓
Cl. 6: $b, \neg e$	$b, \neg e$	$b, \neg e$	$b$	✓
Cl. 7: $\neg d, e$	$\neg d, e$	$\neg d, e$	✓	✓
Cl. 8: $a$	$a$	✓	✓	✓
BCP	$a$	$e$	$b$	SAT
PL	-	-	-	-
Decision	-	-	-	-

5. [Practicals] [3 Points]

Clause 1:  $\{a, b, c\}$

Clause 2:  $\{\neg a, b\}$

Clause 3:  $\{\neg b, c\}$

Clause 4:  $\{\neg c, d\}$

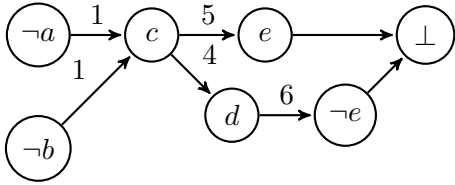
Clause 5:  $\{\neg c, e\}$

Clause 6:  $\{\neg d, \neg e\}$

Solution:



Step	1	2	3	4	5	6
Decision Level	0	1	2	2	2	2
Assignment	-	$\neg a$	$\neg a, \neg b$	$\neg a, \neg b, c$	$\neg a, \neg b, c, d$	$\neg a, \neg b, c, d, e$
Cl. 1: $a, b, c$	$a, b, c$	$b, c$	$c$	✓	✓	✓
Cl. 2: $\neg a, b$	$\neg a, b$	✓	✓	✓	✓	✓
Cl. 3: $\neg b, c$	$\neg b, c$	$\neg b, c$	✓	✓	✓	✓
Cl. 4: $\neg c, d$	$\neg c, d$	$\neg c, d$	$\neg c, d$	$d$	✓	✓
Cl. 5: $\neg c, e$	$\neg c, e$	$\neg c, e$	$\neg c, e$	$e$	$e$	{ } ✗
Cl. 6: $\neg d, \neg e$	$\neg d, \neg e$	$\neg d, \neg e$	$\neg d, \neg e$	$\neg d, \neg e$	$\neg e$	✓
BCP	-	-	$c$	$d$	$\neg e$	-
PL	-	-	-	-	-	-
Decision	$\neg a$	$\neg b$	-	-	-	-

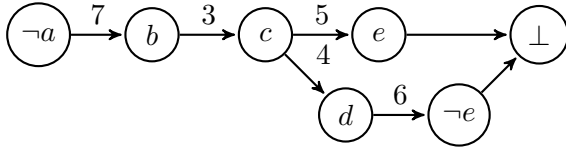


$$\frac{\frac{6. \neg d \vee \neg e}{\neg e \vee \neg c} \quad \frac{4. \neg c \vee d}{\neg c}}{\neg c} \quad \frac{5. \neg c \vee e}{a \vee b} \quad 1. a \vee b \vee c$$

Remark: One could also directly learn  $b$  in this case by further inspection of the clauses:

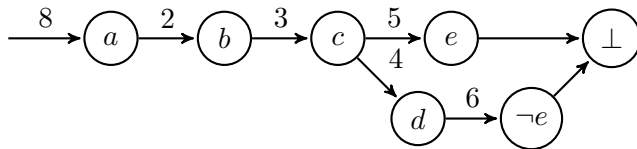
$$\frac{\frac{6. \neg d \vee \neg e}{\neg e \vee \neg c} \quad \frac{4. \neg c \vee d}{\neg c}}{\neg c} \quad \frac{5. \neg c \vee e}{a \vee b} \quad \frac{1. a \vee b \vee c}{b} \quad 2. \neg a \vee b$$

Step	(2)	7	8	9	10
Decision Level	1	1	1	1	1
Assignment	$\neg a$	$\neg a, b$	$\neg a, b, c$	$\neg a, b, c, d$	$\neg a, b, c, d, \neg e$
Cl. 1: $a, b, c$	$b, c$	✓	✓	✓	✓
Cl. 2: $\neg a, b$	✓	✓	✓	✓	✓
Cl. 3: $\neg b, c$	$\neg b, c$	$c$	✓	✓	✓
Cl. 4: $\neg c, d$	$\neg c, d$	$\neg c, d$	$d$	✓	✓
Cl. 5: $\neg c, e$	$\neg c, e$	$\neg c, e$	$e$	$e$	{ } ✗
Cl. 6: $\neg d, \neg e$	$\neg d, \neg e$	$\neg d, \neg e$	$\neg d, \neg e$	$\neg e$	✓
Cl. 7: $a, b$	$b$	✓	✓	✓	✓
BCP	$b$	$c$	$d$	$\neg e$	-
PL	-	-	-	-	-
Decision	-	-	-	-	-



$$\begin{array}{c}
 \frac{6. \neg d \vee \neg e \quad 4. \neg c \vee d}{\neg e \vee \neg c} \quad \frac{5. \neg c \vee e}{\neg c} \quad \frac{3. \neg b \vee c}{\neg b} \quad 7. a \vee b \\
 \hline
 a
 \end{array}$$

Step	(1)	11	12	13	14	15
Decision Level	0	0	0	0	0	0
Assignment	-	$a$	$a, b$	$a, b, c$	$a, b, c, d$	$a, b, c, d, \neg e$
Cl. 1: $a, b, c$	$a, b, c$	✓	✓	✓	✓	✓
Cl. 2: $\neg a, b$	$\neg a, b$	$b$	✓	✓	✓	✓
Cl. 3: $\neg b, c$	$\neg b, c$	$\neg b, c$	$c$	✓	✓	✓
Cl. 4: $\neg c, d$	$\neg c, d$	$\neg c, d$	$\neg c, d$	$d$	✓	✓
Cl. 5: $\neg c, e$	$\neg c, e$	$\neg c, e$	$\neg c, e$	$e$	$e$	{ } ✗
Cl. 6: $\neg d, \neg e$	$\neg d, \neg e$	$\neg d, \neg e$	$\neg d, \neg e$	$\neg e$	$\neg e$	✓
Cl. 7: $a, b$	$a, b$	✓	✓	✓	✓	✓
Cl. 8: $a$	$a$	✓	✓	✓	✓	✓
BCP	$a$	$b$	$c$	$d$	$\neg e$	-
PL	-	-	-	-	-	-
Decision	-	-	-	-	-	UNSAT



$$\begin{array}{c}
 \frac{6. \neg d \vee \neg e \quad 4. \neg c \vee d}{\neg e \vee \neg c} \quad \frac{5. \neg c \vee e}{\neg c} \quad \frac{3. \neg b \vee c}{\neg b} \quad \frac{2. \neg a \vee b}{\neg a} \quad 8. a \\
 \hline
 \perp
 \end{array}$$

6. [Practicals] [4 Points]

Clause 1:  $\{a, \neg c, \neg e\}$

Clause 2:  $\{\neg a, \neg e\}$

Clause 3:  $\{b, e\}$

Clause 4:  $\{\neg b, d, e\}$

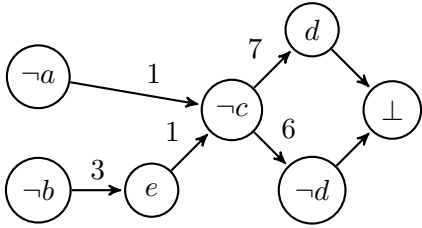
Clause 5:  $\{\neg b, \neg d\}$

Clause 6:  $\{c, \neg d\}$

Clause 7:  $\{c, d\}$

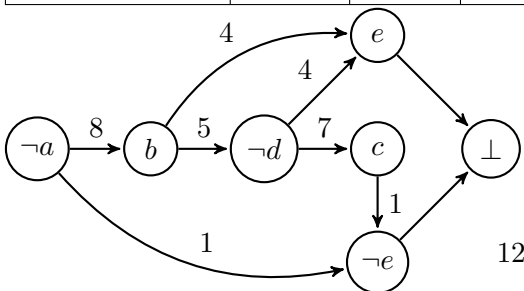
Solution:

Step	1	2	3	4	5	6
Decision Level	0	1	2	2	2	2
Assignment	-	$\neg a$	$\neg a, \neg b$	$\neg a, \neg b, e$	$\neg a, \neg b, e,$ $\neg c$	$\neg a, \neg b, e,$ $\neg c, \neg d$
Cl. 1: $a, \neg c, \neg e$	$a, \neg c, \neg e$	$\neg c, \neg e$	$\neg c, \neg e$	$\neg c$	✓	✓
Cl. 2: $\neg a, \neg e$	$\neg a, \neg e$	✓	✓	✓	✓	✓
Cl. 3: $b, e$	$b, e$	$b, e$	$e$	✓	✓	✓
Cl. 4: $\neg b, d, e$	$\neg b, d, e$	$\neg b, d, e$	✓	✓	✓	✓
Cl. 5: $\neg b, \neg d$	$\neg b, \neg d$	$\neg b, \neg d$	✓	✓	✓	✓
Cl. 6: $c, \neg d$	$c, \neg d$	$c, \neg d$	$c, \neg d$	$c, \neg d$	$\neg d$	✓
Cl. 7: $c, d$	$c, d$	$c, d$	$c, d$	$c, d$	$d$	{ } ✗
BCP	-	-	$e$	$\neg c$	$\neg d$	-
PL	-	-	-	-	-	-
Decision	$\neg a$	$\neg b$	-	-	-	-



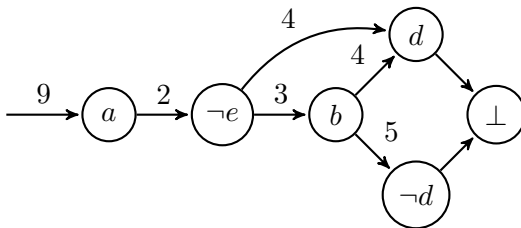
$$\frac{6. \ c \vee \neg d}{c} \quad \frac{7. \ c \vee d}{c} \quad \frac{1. \ a \vee \neg c \vee \neg e}{a \vee \neg e} \quad \frac{3. \ b \vee e}{a \vee b}$$

Step	(2)	7	8	9	10
Decision Level	1	1	1	1	1
Assignment	$\neg a$	$\neg a, b$	$\neg a, b, \neg d$	$\neg a, b, \neg d,$ $c$	$\neg a, b, \neg d,$ $c, \neg e$
Cl. 1: $a, \neg c, \neg e$	$\neg c, \neg e$	$\neg c, \neg e$	$\neg c, \neg e$	$\neg e$	✓
Cl. 2: $\neg a, \neg e$	✓	✓	✓	✓	✓
Cl. 3: $b, e$	$b, e$	✓	✓	✓	✓
Cl. 4: $\neg b, d, e$	$\neg b, d, e$	$d, e$	$e$	$e$	{ } ✗
Cl. 5: $\neg b, \neg d$	$\neg b, \neg d$	$\neg d$	✓	✓	✓
Cl. 6: $c, \neg d$	$c, \neg d$	$c, \neg d$	✓	✓	✓
Cl. 7: $c, d$	$c, d$	$c, d$	$c$	✓	✓
Cl. 8: $a, b$	$b$	✓	✓	✓	✓
BCP	$b$	$\neg d$	$c$	$\neg e$	-
PL	-	-	-	-	-
Decision	-	-	-	-	-



$$\begin{array}{c}
 \frac{1. a \vee \neg c \vee \neg e \quad 4. \neg b \vee d \vee \vee e}{a \vee \neg b \vee \neg c \vee d} \quad 7. c \vee d \\
 \frac{a \vee \neg b \vee d}{a \vee \neg b} \quad 5. \neg b \vee \neg d \\
 \frac{a \vee \neg b}{a} \quad 8. a \vee b
 \end{array}$$

Step	(1)	11	12	13	14
Decision Level	0	0	0	0	0
Assignment	-	$a$	$a, \neg e$	$a, \neg e, b$	$a, \neg e, b, \neg d$
Cl. 1: $a, \neg c, \neg e$	$a, \neg c, \neg e$	✓	✓	✓	✓
Cl. 2: $\neg a, \neg e$	$\neg a, \neg e$	$\neg e$	✓	✓	✓
Cl. 3: $b, e$	$b, e$	$b, e$	$b$	✓	✓
Cl. 4: $\neg b, d, e$	$\neg b, d, e$	$\neg b, d, e$	$\neg b, d$	$d$	$\{\} \times$
Cl. 5: $\neg b, \neg d$	$\neg b, \neg d$	$\neg b, \neg d$	$\neg b, \neg d$	$\neg d$	✓
Cl. 6: $c, \neg d$	$c, \neg d$	$c, \neg d$	$c, \neg d$	$c, \neg d$	✓
Cl. 7: $c, d$	$c, d$	$c, d$	$c, d$	$c, d$	$c$
Cl. 8: $a, b$	$a, b$	✓	✓	✓	✓
Cl. 9: $a$	$a$	✓	✓	✓	✓
BCP	$a$	$\neg e$	$b$	$\neg d$	-
PL	-	-	-	-	-
Decision	-	-	-	-	UNSAT



$$\begin{array}{c}
 \frac{5. \neg b \vee \neg d \quad 4. \neg b \vee d \vee \vee e}{\neg b \vee e} \quad 3. b \vee e \\
 \frac{\neg b \vee e}{e} \quad 2. \neg a \vee \neg e \\
 \frac{e}{\neg a} \quad 8. a \\
 \perp
 \end{array}$$

7. [Practicals] [3 Points]

You are about to plan a train journey in Europe, but you are not yet sure, where to go. You have a few cities in mind, but there are a few restrictions due to a pandemic:

Your biggest wish is to go to Paris, you are definitely going there. After visiting Paris you are either going to London, or to Madrid, but not both. There is no direct train from your home to Paris, therefore you can take a train either via Berlin or via Zurich. On your way back you can choose between Amsterdam or Zurich. As you want to visit as many cities as possible, you do not want to go through Zurich twice, therefore you have to go at least through once through Amsterdam or Berlin. As traveling is currently restricted due to a pandemic, you may not visit Madrid after you visited Berlin and vice versa. You may also not visit London after you went to Amsterdam and vice versa.

Create a CNF from this description. You can use the following rule to make the formula shorter:

$$(\neg s \wedge t) \vee (s \wedge \neg t) \vdash \neg s \vee \neg t$$

Then use the DPLL algorithm to figure out which which cities would be theoretically possible to visit during the vacation. Formulate your answer as a sentence in English.

Solution:

There is no solution available for this question yet.