Logic and Computability SS22
Assignment Sheet 1

Practical Session: 01. 04. 2022

For each of the following sequents, either provide a natural deduction proof, or a counter-example that proves the sequent invalid.

For proofs, clearly indicate which rule, and what assumptions/premises/
intermediate results you are using in each step. Also clearly indicate the scope of any boxes you use.

For counterexamples, give a complete model. Show that the model satisfies the premise(s) of the sequent in question, but does not satisfy the respective conclusion. For each of the following sequents, either provide a natural deduction proof, or a counter-example that proves the sequent invalid.

1. [Practicals] **[2 Points]**

   (a) If I am ill, I go to the doctor.
   I am ill.
   Therefore, I go to the doctor.

   (b) If I am ill, I go to the doctor.
   I go to the doctor.
   Therefore, I am ill.

   (c) (Solve without using the Modus Tollens)
   If I am ill, I go to the doctor.
   I did not go to the doctor.
   Therefore, I am not ill.

2. [Practicals] **[2 Points]**

   (a) $(p \land q) \land \neg r \vdash q \lor r$
   (b) $(p \lor q) \land \neg r \vdash q \land r$

3. [Practicals] **[2 Points]**

   (a) $\vdash (p \to q) \to p$
   (b) $\vdash p \to (q \to p)$

4. [Practicals] **[2 Points]** $\neg(a \land b) \lor \neg c \vdash \neg(a \land b) \to c \lor a$

5. [Practicals] **[2 Points]** $p \land q \lor r \vdash (p \lor r) \land (q \lor r)$

6. [Practicals] **[2 Points]** $\neg\neg x \to \neg y \land z \vdash z \to \neg x \land \neg\neg y$

7. [Practicals] **[2 Points]** $\vdash \neg(p \wedge q) \vee p$
8. [Practicals] **[2 Points]** $\neg(a \vee b) \vdash \neg a \wedge \neg b$
9. [Practicals] **[2 Points]** $(s \vee \neg u) \rightarrow t \vdash (\neg s \wedge u) \vee t$
10. [Practicals] **[2 Points]** $\neg\neg k \rightarrow (l \vee m), \neg\neg\neg l \rightarrow m \vdash \neg k \vee (l \vee \neg\neg m)$