Logic and Computability SS21, Assignment 4

Deadline: 2021-06-04 3:59am

For each of the following sequents, either provide a natural deduction proof, or a counter-example that proves the sequent invalid.
For proofs, clearly indicate which rule, and what assumptions/premises/ intermediate results you are using in each step. Also clearly indicate the scope of any boxes you use.
For counterexamples, give a complete model. Show that the model satisfies the premise(s) of the sequent in question, but does not satisfy the respective conclusion.

20. **[1 Point]** $\forall x P(x) \land \forall x Q(x) \ \vdash \ \forall y\big(P(y) \lor Q(y)\big)$
21. **[1 Point]** $\exists x P(x) \land \exists x Q(x) \ \vdash \ \exists y\big(P(y) \lor Q(y)\big)$
22. **[1 Point]** $\exists x\big(\neg P(x) \lor Q(x)\big) \ \vdash \ \forall x\big(P(x) \to Q(x)\big)$
23. **[1 Point]** $\forall x\big(\neg P(x) \lor Q(x)\big) \ \vdash \ \exists x\big(P(x) \to Q(x)\big)$
24. **[1 Point]** $\forall x P(x) \to Q(x, y), \exists x \exists y\, Q(x, y) \ \vdash \ \exists x\, \neg P(x)$
25. **[1 Point]** $\forall x \exists y\big(f(x) = y\big) \ \vdash \ \forall y \exists x\big(f(x) = y\big)$
26. **[1 Point]** $\exists x \forall y\, \neg P(x, y) \ \vdash \ \forall y \exists x\, \neg\big(P(x, y) \land Q(y, x)\big)$
27. **[1 Point]** $\exists x\big(P(x) \land \neg Q(y)\big) \ \vdash \ \neg \forall x\big(P(x) \to Q(y)\big)$
28. **[2 Points]** $\exists x\big(f(x) = z\big), \forall y\big(f(y) = z\big) \ \vdash \ \forall g \exists h\big(f(g) = f(h)\big)$
29. **[2 Points Bonus]** $\neg\big(y > (x + x) \lor x = 2\big) \to \neg(y = y)$
$\vdash \ z = z \land \big((x + x) = z\big) \to \big((2 + 2) = z \lor y > z\big)$