Logic and Computability SS21, Assignment 5

June 18, 2021

30. Eager encoding uses Ackermann's reduction to get rid of function instances. Perform Ackermann's reduction and show the resulting formulas if you want to check for satisfiability and validity respectively for the following formulas from $\mathcal{T}_{UE}$.

    (a) **[1 Point]**
    $f(x) = f(y) \wedge f(y) = y \vee$
    $f(g(x)) = f(f(y)) \wedge g(x) = x \vee$
    $f(x) \neq f(y) \wedge y \neq g(f(y)) \wedge x \neq g(x)$

    (b) **[2 Point]**
    $(f(x, z) = x \leftrightarrow f(x, y) = x) \wedge$
    $(y = z \vee f(x, x) = f(x, y)) \rightarrow f(x, z) = z$

31. The second step in eager encoding of $\mathcal{T}_{UE}$is a graph-based reduction. Perform this reduction on the formulas from the previous example.

    (a) **[1 Point]** Formula from 30a.

    (b) **[2 Point]** Formula from 30b.

32. Use the Congruence-Closure algorithm to check if the following assignment for the equalities is satisfiable.

    (a) **[1 Point]**
    $f(a) \neq c \wedge f(a) = f(b) \wedge a = f(a) \wedge$
    $f(b) = c \wedge a \neq b \wedge f(a) = b$

    (b) **[1 Point]**
    $f(a) = c \wedge f(c) \neq f(d) \wedge b = f(c) \wedge$
    $a \neq f(c) \wedge c = d \wedge b \neq d \wedge a = c$

    (c) **[2 Point]**
    $a = b \wedge c \neq d \wedge f(a) = c \wedge$
    $f(b) \neq f(c) \wedge f(a) = f(d) \wedge$
    $f(b) = c \wedge f(d) = f(c)$