

Logic and Computability SS21,
Assignment 2,
Solution

Deadline: 2021-04-30 3:59am

1. [1 Points]

$$Cl.1 = \{\neg c, e\}$$

$$Cl.2 = \{\neg a, c\}$$

$$Cl.3 = \{\neg c, d\}$$

$$Cl.4 = \{a, \neg d\}$$

$$Cl.5 = \{\neg b, c\}$$

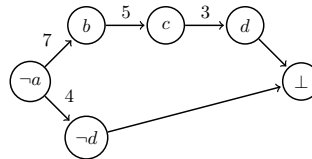
$$Cl.6 = \{d, \neg e\}$$

$$Cl.7 = \{a, b\}$$

Solution:

Step	1	2	3	4	5	6(1)	7	8	9	10
Decision Level	0	1	1	1	1	0	0	0	0	0
Assignment	-	$\neg a$	$\neg a, b$	$\neg a, b, c$	$\neg a, b, c, \neg d$	-	a	a, c	a, c, d	a, c, d, e
Cl. 1: $\neg c, e$	$\neg c, e$	$\neg c, e$	$\neg c, e$	e	e	$\neg c, e$	$\neg c, e$	e	e	✓
Cl. 2: $\neg a, c$	$\neg a, c$	✓	✓	✓	✓	$\neg a, c$	c	✓	✓	✓
Cl. 3: $\neg c, d$	$\neg c, d$	$\neg c, d$	$\neg c, d$	d	{ } ✗	$\neg c, d$	$\neg c, d$	d	✓	✓
Cl. 4: a, $\neg d$	a, $\neg d$	$\neg d$	$\neg d$	$\neg d$	✓	a, $\neg d$	✓	✓	✓	✓
Cl. 5: $\neg b, c$	$\neg b, c$	$\neg b, c$	c	✓	✓	$\neg b, c$	$\neg b, c$	✓	✓	✓
Cl. 6: d, $\neg e$	d, $\neg e$	d, $\neg e$	d, $\neg e$	d, $\neg e$	$\neg e$	d, $\neg e$	d, $\neg e$	d, $\neg e$	✓	✓
Cl. 7: a, b	a, b	b	✓	✓	✓	a, b	✓	✓	✓	✓
Cl. 8: a (LC)	-	-	-	-	-	a	✓	✓	✓	✓
BCP	-	b	c	$\neg d$	-	a	c	d	e	-
PL	-	-	-	-	-	-	-	-	-	-
Decision	$\neg a$	-	-	-	-	-	-	-	-	SAT

ad 5:



$$\mathcal{M} : a, b, c, d, e$$

or

$$\mathcal{M} : a, \neg b, c, d, e$$

$$\frac{3. \neg c \vee d \quad 4. a \vee \neg d}{a \vee \neg c} \quad \frac{5. \neg b \vee c}{a \vee \neg b} \quad \frac{7. a \vee b}{a}$$

2. [1 Point]

$$Cl.1 = \{\neg a, \neg b\}$$

$$Cl.2 = \{\neg c, \neg d\}$$

$$Cl.3 = \{c, \neg e\}$$

$$Cl.4 = \{a, c\}$$

$$Cl.5 = \{\neg b, d\}$$

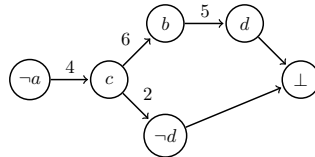
$$Cl.6 = \{b, \neg c\}$$

$$Cl.7 = \{c, e\}$$

Solution:

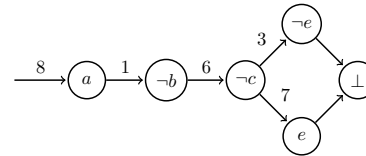
Step	1	2	3	4	5	6(1)	7	8	9	10
Decision Level	0	1	1	1	1	0	0	0	0	0
Assignment	-	$\neg a$	$\neg a, c$	$\neg a, b, c$	$\neg a, b, c, \neg d$	-	a	$a, \neg b$	$a, \neg b, \neg c$	$a, \neg b, \neg c, \neg e$
Cl. 1: $\neg a, \neg b$	$\neg a, \neg b$	✓	✓	✓	✓	$\neg a, \neg b$	$\neg b$	✓	✓	✓
Cl. 2: $\neg c, \neg d$	$\neg c, \neg d$	$\neg c, \neg d$	$\neg d$	$\neg d$	✓	$\neg c, \neg d$	$\neg c, \neg d$	$\neg c, \neg d$	✓	✓
Cl. 3: $c, \neg e$	$c, \neg e$	$c, \neg e$	✓	✓	✓	$c, \neg e$	$c, \neg e$	$c, \neg e$	$\neg e$	✓
Cl. 4: a, c	a, c	c	✓	✓	✓	a, c	✓	✓	✓	✓
Cl. 5: $\neg b, d$	$\neg b, d$	$\neg b, d$	$\neg b, d$	d	$\{\}$ ✗	$\neg b, d$	$\neg b, d$	$\neg b, d$	✓	✓
Cl. 6: $b, \neg c$	$b, \neg c$	$b, \neg c$	b	✓	✓	$b, \neg c$	$b, \neg c$	$\neg c$	✓	✓
Cl. 7: c, e	c, e	c, e	✓	✓	✓	c, e	c, e	c, e	e	$\{\}$ ✗
Cl. 8: a (LC)	-	-	-	-	-	a	✓	✓	✓	✓
BCP	-	c	b	$\neg d$	-	a	$\neg b$	$\neg c$	$\neg e$	-
PL	-	-	-	-	-	-	-	-	-	-
Decision	$\neg a$	-	-	-	-	-	-	-	-	UNSAT

ad 5:



$$\begin{array}{l}
 \frac{2. \neg c \vee \neg d}{\neg b \vee \neg c} \quad \frac{5. \neg b \vee d}{\neg c} \quad \frac{6. b \vee \neg c}{a} \quad 4. a \vee c
 \end{array}$$

ad 10:



$$\begin{array}{l}
 \frac{3. \neg c \vee \neg e}{c} \quad \frac{7. c \vee e}{b} \quad \frac{6. b \vee \neg c}{\neg a} \quad \frac{1. \neg a \vee \neg b}{\perp} \quad 8. a
 \end{array}$$

3. [1 Point]

$$Cl.1 = \{\neg a, \neg b\}$$

$$Cl.2 = \{a, d, e\}$$

$$Cl.3 = \{b, \neg c\}$$

$$Cl.4 = \{c, \neg d, e\}$$

$$Cl.5 = \{\neg c, e\}$$

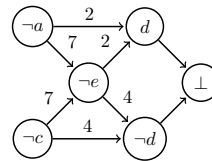
$$Cl.6 = \{\neg a, b\}$$

$$Cl.7 = \{a, c, \neg e\}$$

Solution:

Step	1	2	3	4	5	6	7(1)	8	9	10
Decision Level	0	1	1	2	2	2	1	1	1	1
Assignment	-	$\neg a$	$\neg a, b$	$\neg a, b, \neg c$	$\neg a, b, \neg c, \neg e$	$\neg a, b, \neg c, \neg d, \neg e$	$\neg a$	$\neg a, c$	$\neg a, b, c$	$\neg a, b, c, e$
Cl. 1: $\neg a, \neg b$	$\neg a, \neg b$	✓	✓	✓	✓	✓	$\neg a, \neg b$	✓	✓	✓
Cl. 2: a, d, e	a, d, e	d, e	d, e	d, e	d	$\{\}$ ✗	a, d, e	d, e	d, e	✓
Cl. 3: $b, \neg c$	$b, \neg c$	$b, \neg c$	✓	✓	✓	✓	$b, \neg c$	b	✓	✓
Cl. 4: $c, \neg d, e$	$c, \neg d, e$	$c, \neg d, e$	$c, \neg d, e$	$c, \neg d, e$	$\neg d, e$	$\neg d$	$c, \neg d, e$	✓	✓	✓
Cl. 5: $\neg c, e$	$\neg c, e$	$\neg c, e$	$\neg c, e$	$\neg c, e$	✓	✓	$\neg c, e$	e	e	✓
Cl. 6: $\neg a, b$	$\neg a, b$	✓	✓	✓	✓	✓	$\neg a, b$	✓	✓	✓
Cl. 7: $a, c, \neg e$	$a, c, \neg e$	$c, \neg e$	$c, \neg e$	$\neg e$	✓	✓	$a, c, \neg e$	✓	✓	✓
BCP	-	-	-	$\neg e$	$\neg d$	-	c	b	e	-
PL	-	b	-	-	-	-	-	-	-	-
Decision	$\neg a$	-	$\neg c$	-	-	-	-	-	-	SAT

ad 6:



$$\mathcal{M} : \neg a, b, c, d, e$$

or

$$\mathcal{M} : \neg a, b, c, \neg d, e$$

$$\frac{2. a \vee d \vee e \quad 4. c \vee \neg d \vee e}{a \vee c \vee e} \quad \frac{7. a \vee c \vee \neg e}{a \vee c}$$

4. [2 Points]

$$Cl.1 = \{\neg b, c, d\}$$

$$Cl.2 = \{\neg b, \neg d\}$$

$$Cl.3 = \{a, \neg c\}$$

$$Cl.4 = \{\neg c, e\}$$

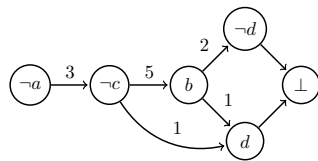
$$Cl.5 = \{b, c\}$$

$$Cl.6 = \{\neg a, \neg e\}$$

Solution:

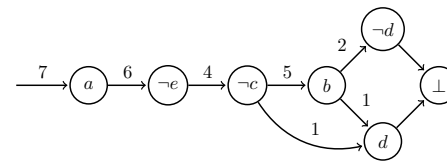
Step	1	2	3	4	5	6(1)	7	8	9	10	11
Decision Level	0	1	1	1	1	1	0	0	0	0	0
Assignment	-	$\neg a$	$\neg a, \neg c$	$\neg a, b, \neg c$	$\neg a, b, \neg c, \neg d$	-	a	$a, \neg e$	$a, \neg c, \neg e$	$a, b, \neg c, \neg e$	$a, b, \neg c, \neg d, \neg e$
Cl. 1: $\neg b, c, d$	$\neg b, c, d$	$\neg b, c, d$	$\neg b, d$	d	$\{\} \times$	$\neg b, c, d$	$\neg b, c, d$	$\neg b, c, d$	$\neg b, d$	d	$\{\} \times$
Cl. 2: $\neg b, \neg d$	$\neg b, \neg d$	$\neg b, \neg d$	$\neg b, \neg d$	$\neg d$	\checkmark	$\neg b, \neg d$	$\neg b, \neg d$	$\neg b, \neg d$	$\neg b, \neg d$	$\neg d$	\checkmark
Cl. 3: $a, \neg c$	$a, \neg c$	$\neg c$	\checkmark	\checkmark	\checkmark	$a, \neg c$	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
Cl. 4: $\neg c, e$	$\neg c, e$	$\neg c, e$	\checkmark	\checkmark	\checkmark	$\neg c, e$	$\neg c, e$	$\neg c$	\checkmark	\checkmark	\checkmark
Cl. 5: b, c	b, c	b, c	b	\checkmark	\checkmark	b, c	b, c	b, c	b	\checkmark	\checkmark
Cl. 6: $\neg a, \neg e$	$\neg a, \neg e$	\checkmark	\checkmark	\checkmark	\checkmark	$\neg a, \neg e$	$\neg e$	\checkmark	\checkmark	\checkmark	\checkmark
Cl. 7: a (LC)	-	-	-	-	-	a	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
BCP	-	$\neg c$	b	$\neg d$	-	a	$\neg e$	$\neg c$	b	$\neg d$	-
PL	-	-	-	-	-	-	-	-	-	-	-
Decision	$\neg a$	-	-	-	-	-	-	-	-	-	UNSAT

ad 5:



$$\frac{\frac{\frac{1. \neg b \vee c \vee d}{\neg b \vee c} \quad 2. \neg b \vee \neg d}{c} \quad 5. b \vee c}{a} \quad 3. a \vee \neg c$$

ad 12:



$$\frac{\frac{\frac{1. \neg b \vee c \vee d}{\neg b \vee c} \quad 2. \neg b \vee \neg d}{c} \quad 5. b \vee c}{e} \quad 4. \neg c \vee e}{a} \quad 6. \neg a \vee \neg e}{\perp} \quad 7. \neg a$$

5. [2 Points]

$$Cl.1 = \{\neg c, \neg e\}$$

$$Cl.2 = \{\neg d, e\}$$

$$Cl.3 = \{a, b, c\}$$

$$Cl.4 = \{\neg a, \neg d\}$$

$$Cl.5 = \{d, e\}$$

$$Cl.6 = \{a, \neg c, \neg d\}$$

$$Cl.7 = \{\neg b, c\}$$

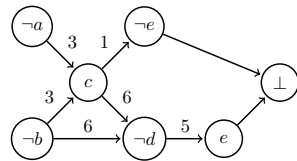
Solution:

Step	1	2	3	4	5	6	7(2)	8	9	10	11
Decision Level	0	1	2	2	2	2	1	1	1	1	1
Assignment	-	$\neg a$	$\neg a, \neg b$	$\neg a, \neg b, c$	$\neg a, \neg b, c, \neg d$	$\neg a, \neg b, c, \neg d, \neg e$	$\neg a$	$\neg a, b$	$\neg a, b, c$	$\neg a, b, c, \neg d$	$\neg a, b, c, \neg d, \neg e$
Cl. 1: $\neg c, \neg e$	$\neg c, \neg e$	$\neg c, \neg e$	$\neg c, \neg e$	$\neg e$	$\neg e$	✓	$\neg c, \neg e$	$\neg c, \neg e$	$\neg e$	$\neg e$	✓
Cl. 2: $\neg d, e$	$\neg d, e$	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Cl. 3: a, b, c	a, b, c	b, c	c	✓	✓	✓	✓	✓	✓	✓	✓
Cl. 4: $\neg a, \neg d$	$\neg a, \neg d$	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Cl. 5: d, e	d, e	d, e	d, e	d, e	e	{ } ✗	d, e	d, e	d, e	e	{ } ✗
Cl. 6: $a, \neg c, \neg d$	$a, \neg c, \neg d$	$\neg c, \neg d$	$\neg c, \neg d$	$\neg d$	✓	✓	$\neg c, \neg d$	$\neg c, \neg d$	$\neg d$	✓	✓
Cl. 7: $\neg b, c$	$\neg b, c$	$\neg b, c$	✓	✓	✓	✓	$\neg b, c$	c	✓	✓	✓
Cl. 8: a, b (LC)	-	-	-	-	-	-	b	✓	✓	✓	✓
Cl. 9: a (LC)	-	-	-	-	-	-	-	-	-	-	-
BCP	-	-	c	$\neg d$	$\neg e$	-	b	c	$\neg d$	$\neg e$	-
PL	-	-	-	-	-	-	-	-	-	-	-
Decision	$\neg a$	$\neg b$	-	-	-	-	-	-	-	-	-

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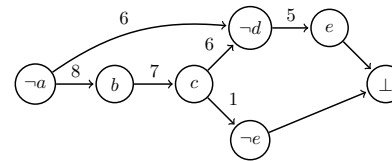
Step	12(1)	13	14	15	16	17
Decision Level	0	0	0	0	0	0
Assignment	-	a	a, $\neg d$	a, $\neg d$, e	a, $\neg c$, $\neg d$, e	a, $\neg b$, $\neg c$, $\neg d$, e
Cl. 1: $\neg c, \neg e$	$\neg c, \neg e$	$\neg c, \neg e$	$\neg c, \neg e$	$\neg c$	✓	✓
Cl. 2: $\neg d, e$	$\neg d, e$	e	e	✓	✓	✓
Cl. 3: a, b, c	a, b, c	✓	✓	✓	✓	✓
Cl. 4: $\neg a, \neg d$	$\neg a, \neg d$	$\neg d$	✓	✓	✓	✓
Cl. 5: d, e	d, e	d, e	e	✓	✓	✓
Cl. 6: a, $\neg c, \neg d$	a, $\neg c, \neg d$	✓	✓	✓	✓	✓
Cl. 7: $\neg b, c$	$\neg b, c$	$\neg b, c$	$\neg b, c$	$\neg b, c$	$\neg b$	✓
Cl. 8: a, b (LC)	a, b	✓	✓	✓	✓	✓
Cl. 9: a (LC)	a	✓	✓	✓	✓	✓
BCP	a	$\neg d$	e	$\neg c$	$\neg b$	-
PL	-	-	-	-	-	-
Decision	-	-	-	-	-	SAT

ad 6:



$$\begin{array}{l}
 \frac{1. \neg c \vee \neg e \quad 5. d \vee e}{\neg c \vee d} \quad \frac{6. a \vee \neg c \vee \neg d}{a \vee \neg c} \quad 3. a \vee b \vee c \\
 \hline
 a \vee b
 \end{array}$$

ad 11:



$$\begin{array}{l}
 \frac{1. \neg c \vee \neg e \quad 5. d \vee e}{\neg c \vee d} \quad \frac{6. a \vee \neg c \vee \neg d}{a \vee \neg c} \quad \frac{7. \neg b \vee c}{a \vee \neg b} \quad 8. a \vee b \\
 \hline
 a
 \end{array}$$

$$M : a, \neg b, \neg c, \neg d, e$$

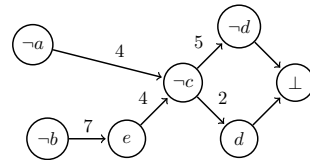
6. [2 Points]

- $Cl.1 = \{\neg b, d, e\}$
- $Cl.2 = \{c, d\}$
- $Cl.3 = \{\neg a, \neg e\}$
- $Cl.4 = \{a, \neg c, \neg e\}$
- $Cl.5 = \{c, \neg d\}$
- $Cl.6 = \{\neg b, \neg d\}$
- $Cl.7 = \{b, e\}$

Solution:

Step	1	2	3	4	5	6
Decision Level	0	1	2	2	2	2
Assignment	-	$\neg a$	$\neg a, \neg b$	$\neg a, \neg b, e$	$\neg a, \neg b, \neg c, e$	$\neg a, \neg b, \neg c, \neg d, e$
Cl. 1: b, d, e	$\neg b, d, e$	$\neg b, d, e$	✓	✓	✓	✓
Cl. 2: c, d	c, d	c, d	c, d	c, d	d	{ } ✗
Cl. 3: $\neg a, \neg e$	$\neg a, \neg e$	✓	✓	✓	✓	✓
Cl. 4: a, $\neg c, \neg e$	a, $\neg c, \neg e$	$\neg c, \neg e$	$\neg c, \neg e$	$\neg c$	✓	✓
Cl. 5: c, $\neg d$	c, $\neg d$	c, $\neg d$	c, $\neg d$	c, $\neg d$	$\neg d$	✓
Cl. 6: $\neg b, \neg d$	$\neg b, \neg d$	$\neg b, \neg d$	✓	✓	✓	✓
Cl. 7: b, e	b, e	b, e	e	✓	✓	✓
Cl. 8: a, b (LC)	-	-	-	-	-	-
Cl. 9: a (LC)	-	-	-	-	-	-
BCP	-	-	e	$\neg c$	$\neg d$	-
PL	-	-	-	-	-	-
Decision	$\neg a$	$\neg b$	-	-	-	-

ad 6:

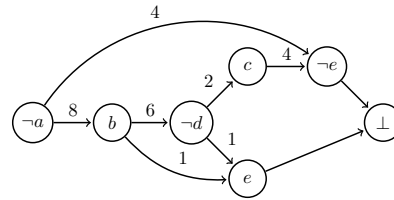


Solution continues on next page.

$$\begin{array}{c}
 \frac{2. c \vee d}{c} \quad \frac{5. c \vee \neg d}{c} \\
 \hline
 \frac{4. a \vee \neg c \vee \neg e}{a \vee \neg e} \quad \frac{7. b \vee e}{a \vee b}
 \end{array}$$

Step	7(2)	8	9	20	11	12(1)	13	14	15	16
Decision Level	1	1	1	1	1	0	0	0	0	0
Assignment	$\neg a$	$\neg a, b$	$\neg a, b, \neg d$	$\neg a, b, c, \neg d$	$\neg a, b, c, \neg d, \neg e$	-	a	$a, \neg e$	$a, b, \neg e$	$a, b, \neg d, \neg e$
Cl. 1: $\neg b, d, e$	$\neg b, d, e$	$\neg b, d, e$	d, e	e	$\{\} \times$	$\neg b, d, e$	$\neg b, d, e$	$\neg b, d, e$	$\neg b, d$	$\{\} \times$
Cl. 2: c, d	c, d	c, d	c	✓	✓	c, d	c, d	c, d	c, d	c
Cl. 3: $\neg a, \neg e$	✓	✓	✓	✓	✓	$\neg a, \neg e$	$\neg e$	✓	✓	✓
Cl. 4: $a, \neg c, \neg e$	$\neg c, \neg e$	$\neg c, \neg e$	$\neg c, \neg e$	$\neg e$	✓	$a, \neg c, \neg e$	✓	✓	✓	✓
Cl. 5: $c, \neg d$	c, $\neg d$	c, $\neg d$	✓	✓	✓	c, $\neg d$	c, $\neg d$	c, $\neg d$	c, $\neg d$	✓
Cl. 6: $\neg b, \neg d$	$\neg b, \neg d$	$\neg d$	✓	✓	✓	$\neg b, \neg d$	$\neg b, \neg d$	$\neg b, \neg d$	$\neg d$	✓
Cl. 7: b, e	b, e	✓	✓	✓	✓	b, e	b, e	b	✓	✓
Cl. 8: a, b (LC)	b	✓	✓	✓	✓	a, b	✓	✓	✓	✓
Cl. 9: a (LC)	-	-	-	-	-	a	✓	✓	✓	✓
BCP	b	$\neg d$	c	$\neg e$	-	a	$\neg e$	b	$\neg d$	-
PL	-	-	-	-	-	-	-	-	-	-
Decision	-	-	-	-	-	-	-	-	-	UNSAT

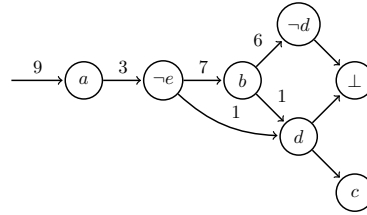
ad 11:



$$\begin{array}{r}
 \frac{1. \neg b \vee d \vee e \quad 4. a \vee \neg c \vee \neg e}{a \vee \neg b \vee \neg c \vee d} \quad 2. c \vee d \\
 \frac{a \vee \neg b \vee d}{a \vee \neg b} \quad 6. \neg b \vee \neg d \quad 8. a \vee b \\
 \hline
 a
 \end{array}$$

Solution continues on next page.

ad 16:



$$\begin{array}{r}
 \frac{1. \neg b \vee d \vee e}{\neg b \vee e} \quad \frac{6. \neg b \vee \neg d}{e} \quad \frac{7. b \vee e}{\neg a} \quad \frac{3. \neg a \vee \neg e}{\perp} \quad \frac{9. a}{\perp}
 \end{array}$$

7. [2 Points] It is lunchtime on a Sunday and your fridge is almost empty. You think that you can probably prepare a decent pizza with the little ingredients you have.

You do have dough. The dough is absolutely necessary for your pizza. You also have arugula, bell pepper and eggplant. You want to put at least one of those three ingredients as toppings on your pizza. Cheese is necessary for the pizza too. You have cheddar and feta. You can use one or both kinds of cheese. You don't like the combination of feta and bell pepper, so you can put at most one of those two ingredients on your pizza. Furthermore you need to save some veggies for dinner, so you can only use either the bell pepper or the eggplant for your pizza.

Create a CNF from this description. You can use the following rule to make the formula shorter (You can try to prove this formula using natural deduction to get some additional practice.):

$$(\neg s \wedge t) \vee (s \wedge \neg t) \vdash \neg s \vee \neg t$$

Then use a DPLL to figure out which ingredients you should use for your pizza and which ingredients you shouldn't use. Formulate your answer as a sentence.

Solution:

CNF: $\text{dough} \wedge (\text{arugula} \vee \text{bell pepper} \vee \text{eggplant}) \wedge (\text{cheddar cheese} \vee \text{feta cheese}) \wedge (\neg \text{b} \vee \neg \text{f}) \wedge (\neg \text{b} \vee \neg \text{e})$

Solution:

Step	1	2	3	4	5
Decision Level	0	0	0	0	0
Assignment	-	d	a, d	a, \neg b, d	a, \neg b, c, d
Cl. 1: d	d	✓	✓	✓	✓
Cl. 2: a, b, e	a, b, e	a, b, e	✓	✓	✓
Cl. 3: c, f	c, f	c, f	c, f	c, f	✓
Cl. 4: \neg b, \neg f	\neg b, \neg f	\neg b, \neg f	\neg b, \neg f	✓	✓
Cl. 5: \neg b, \neg e	\neg b, \neg e	\neg b, \neg e	\neg b, \neg e	✓	✓
BCP	d	-	-	-	-
PL	-	a	\neg b	c	-
Decision	-	-	-	-	SAT

I should use arugula, cheddar cheese and dough.

I should not use bell pepper.

I may or may not use eggplant and feta cheese.

$$(\neg s \wedge t) \vee (s \wedge \neg t) \vdash \neg s \vee \neg t$$

1.	$(\neg s \wedge t) \vee (s \wedge \neg t)$	prem
2.	$\neg s \wedge t$	ass
3.	$\neg s$	e_1 2
4.	$\neg s \vee t$	i_1 3
5.	$s \wedge \neg t$	ass
6.	$\neg t$	e_2 5
7.	$\neg s \vee t$	i_2 6
8.	$\neg s \vee t$	e 1, 2-4, 5-7