

Logic and Computability SS21,
Assignment 1,
Solution

Deadline: 2021-04-16 3:59am

1. [1 Point]

- (a)
 - If I wear a mask and sanitize my hands regularly, I will not get infected and I will not infect anyone else.
 - I wear a mask.
 - I do not sanitize my hands regularly.
 - *Therefore*, I will get infected.
- (b)
 - If I wear a mask and sanitize my hands regularly, I will not get infected and I will not infect anyone else.
 - I will not get infected and I will not infect anyone else.
 - *Therefore*, I wear a mask.
- (c)
 - If I wear a mask and sanitize my hands regularly, I will not get infected and I will not infect anyone else.
 - I wear a mask and sanitize my hands regularly.
 - *Therefore*, I will not infect anyone else.

Solution:

wear a mask.....m
sanitize hands regularly s
get infected i
infect others.....o

(a) $(m \wedge s) \rightarrow (\neg i \wedge \neg o), m, \neg s \vdash i$

$m \dots\dots\dots \top$
Model M : $s \dots\dots\dots \perp$
 $i \dots\dots\dots \perp$
 $o \dots\dots\dots$ either \top or \perp

$M \models (m \wedge s) \rightarrow (\neg i \wedge \neg o)$
 $M \models m$
 $M \models \neg s$
 $M \not\models i$

$(m \wedge s) \rightarrow (\neg i \wedge \neg o), m, \neg s \not\vdash i$
 M is a counterexample.

(b) $(m \wedge s) \rightarrow (\neg i \wedge \neg o), \neg i \wedge \neg o \vdash m$

$m \dots\dots\dots \perp$
Model M : $s \dots\dots\dots$ either \top or \perp
 $i \dots\dots\dots \perp$
 $o \dots\dots\dots \perp$

$M \models (m \wedge s) \rightarrow (\neg i \wedge \neg o)$
 $M \models \neg i \wedge \neg o$
 $M \not\models m$

$(m \wedge s) \rightarrow (\neg i \wedge \neg o), \neg i \wedge \neg o \not\vdash m$
 M is a counterexample.

(c) $(m \wedge s) \rightarrow (\neg i \wedge \neg o), m \wedge s \vdash \neg o$

1. $(m \wedge s) \rightarrow (\neg i \wedge \neg o)$ premise
2. $m \wedge s$ premise
3. $\neg i \wedge \neg o$ $\rightarrow e$ 1,2
4. $\neg o$ $\wedge e_2$ 3

2. [1 Point] $\vdash \neg(p \wedge \neg p)$

1.	$p \wedge \neg p$	assumption
2.	p	$\wedge e_1$ 1
3.	$\neg p$	$\wedge e_2$ 1
4.	\perp	$\neg e$ 2,3
5.	$\neg(p \wedge \neg p)$	$\neg i$ 1-4

3. [1 Point] $(p \vee q) \vee r \vdash p \vee (q \vee r)$

1.	$(p \vee q) \vee r$	premise
2.	$(p \vee q)$	assumption
3.	p	assumption
4.	$p \vee (q \vee r)$	$\vee i_1$ 3
5.	q	assumption
6.	$(q \vee r)$	$\vee i_1$ 5
7.	$p \vee (q \vee r)$	$\vee i_2$ 6
8.	$p \vee (q \vee r)$	$\vee e$ 2,3-4,5-7
9.	r	assumption
10.	$q \vee r$	$\vee i_2$ 9
11.	$p \vee (q \vee r)$	$\vee i_2$ 10
12.	$p \vee (q \vee r)$	$\vee e$ 1,2-8,9-11

4. [1 Point] $p \rightarrow q, r \rightarrow s \vdash (p \wedge r) \rightarrow (q \wedge s)$

1.	$p \rightarrow q$	premise
2.	$r \rightarrow s$	premise
3.	$p \wedge r$	assumption
4.	p	$\wedge e_1$ 3
5.	q	$\rightarrow e$ 1,4
6.	r	$\wedge e_2$ 3
7.	s	$\rightarrow e$ 2,6
8.	$q \wedge s$	$\wedge i$ 5,7
9.	$(p \wedge r) \rightarrow (q \wedge s)$	$\rightarrow i$ 3-8

5. [1 Point] $p \wedge (\neg q \rightarrow r) \vdash q \vee r$

1. $p \wedge (\neg q \rightarrow r)$ premise
2. $(\neg q \rightarrow r)$ $\wedge e_2$ 1
3. $(q \vee \neg q)$ LEM
4.

q	assumption
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5.

$q \vee r$	$\vee i_1$ 4
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6.

$\neg q$	assumption
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7.

r	$\rightarrow e$ 2
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8.

$q \vee r$	$\vee i_2$ 7
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9. $q \vee r$ $\vee e$ 3,4-5,6-8

6. [1 Point] $x \rightarrow y \vdash (x \rightarrow (x \wedge y)) \wedge ((x \wedge y) \rightarrow x)$

1. $x \rightarrow y$ premise
2.

x	assumption
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3.

y	$\rightarrow e$ 1,2
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4.

$x \wedge y$	$\wedge i$ 2,3
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5. $x \rightarrow (x \wedge y)$ $\rightarrow i$ 2-4
6.

$x \wedge y$	assumption
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7.

x	$\wedge e_1$ 6
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8. $(x \wedge y) \rightarrow x$ $\rightarrow i$ 6-7
9. $(x \rightarrow (x \wedge y)) \wedge ((x \wedge y) \rightarrow x)$ $\wedge i$ 5,8

7. [1 Point] $x \rightarrow y \vdash (x \rightarrow (x \vee y)) \wedge ((x \vee y) \rightarrow x)$

Model M : $\begin{matrix} x \dots\dots\dots \perp \\ y \dots\dots\dots \top \end{matrix}$

$M \models x \rightarrow y$
 $M \not\models (x \rightarrow (x \vee y)) \wedge ((x \vee y) \rightarrow x)$

$x \rightarrow y \not\models (x \rightarrow (x \vee y)) \wedge ((x \vee y) \rightarrow x)$
 M is a counterexample.

8. [1 Point] $\neg\neg s \rightarrow (\neg\neg t \vee u), \neg t \rightarrow u \vdash \neg s \vee u$

$s \dots\dots\dots \top$
 Model M : $t \dots\dots\dots \top$
 $u \dots\dots\dots \perp$
 $M \models \neg\neg s \rightarrow (\neg\neg t \vee u)$
 $M \models \neg t \rightarrow u$
 $M \not\models \neg s \vee u$

$\neg\neg s \rightarrow (\neg\neg t \vee u), \neg t \rightarrow u \not\vdash \neg s \vee u$
 M is a counterexample.

9. [1 Point] $\vdash (a \rightarrow b) \vee (b \rightarrow c)$

1.	$b \vee \neg b$	LEM
2.	b	assumption
3.	a	assumption
4.	b	copy 2
5.	$a \rightarrow b$	\rightarrow i 3-4
6.	$(a \rightarrow b) \vee (b \rightarrow c)$	\vee i ₁ 5
7.	$\neg b$	assumption
8.	b	assumption
9.	\perp	\neg e 7,8
10.	c	\perp e 9
11.	$b \rightarrow c$	\rightarrow i 8-9
12.	$(a \rightarrow b) \vee (b \rightarrow c)$	\vee i ₂ 11
13.	$(a \rightarrow b) \vee (b \rightarrow c)$	\vee e 1,2-6,7-12

10. [1 Point] $\vdash ((g \rightarrow h) \rightarrow g \wedge h) \rightarrow g$

1.	$(g \rightarrow h) \rightarrow g \wedge h$	assumption
2.	$\neg g$	assumption
3.	g	assumption
4.	\perp	$\neg e$ 2,3
5.	h	$\perp e$ 4
6.	$g \rightarrow h$	$\rightarrow i$ 3-5
7.	$g \wedge h$	$\rightarrow e$ 1,6
8.	g	$\wedge e_1$ 7
9.	\perp	$\neg e$ 2,8
10.	g	PBC 2-9
11.	$((g \rightarrow h) \rightarrow g \wedge h) \rightarrow g$	$\rightarrow i$ 1-10