

Logic and Computability SS21, Assignment 1

Deadline: 2021-04-16 3:59am

For each of the following sequents, either provide a natural deduction proof, or a counter-example that proves the sequent invalid.

For proofs, clearly indicate which rule, and what assumptions/premises/intermediate results you are using in each step. Also clearly indicate the scope of any boxes you use.

For counterexamples, give a complete model. Show that the model satisfies the premise(s) of the sequent in question, but does not satisfy the respective conclusion.

1. [1 Point]

- (a)
 - If I wear a mask and sanitize my hands regularly, I will not get infected and I will not infect anyone else.
 - I wear a mask.
 - I do not sanitize my hands regularly.
 - *Therefore*, I will get infected.
- (b)
 - If I wear a mask and sanitize my hands regularly, I will not get infected and I will not infect anyone else.
 - I will not get infected and I will not infect anyone else.
 - *Therefore*, I wear a mask.
- (c)
 - If I wear a mask and sanitize my hands regularly, I will not get infected and I will not infect anyone else.
 - I wear a mask and sanitize my hands regularly.
 - *Therefore*, I will not infect anyone else.

2. [1 Point] $\vdash \neg(p \wedge \neg p)$
3. [1 Point] $(p \vee q) \vee r \vdash p \vee (q \vee r)$
4. [1 Point] $p \rightarrow q, r \rightarrow s \vdash (p \wedge r) \rightarrow (q \wedge s)$
5. [1 Point] $p \wedge (\neg q \rightarrow r) \vdash q \vee r$
6. [1 Point] $x \rightarrow y \vdash (x \rightarrow (x \wedge y)) \wedge ((x \wedge y) \rightarrow x)$
7. [1 Point] $x \rightarrow y \vdash (x \rightarrow (x \vee y)) \wedge ((x \vee y) \rightarrow x)$
8. [1 Point] $\neg\neg s \rightarrow (\neg\neg t \vee u), \neg t \rightarrow u \vdash \neg s \vee u$
9. [1 Point] $\vdash (a \rightarrow b) \vee (b \rightarrow c)$
10. [1 Point] $\vdash ((g \rightarrow h) \rightarrow g \wedge h) \rightarrow g$