Computer Organization and Networks

Chapter 3: Finite State Machine



http://www.iaik.tugraz.at

Why should you read this primer?

With this set of slides I guide you through the essence of designing finite state machines. With each slide, you get drawn into the topic step by step.

I use very simple examples in order to concentrate on the essence.

I assume that you know...

- what the term "function" means in mathematics.
- that in logic functions we use only 0s and 1s.
- that a logic function can be uniquely described with a truth table.
- that a truth table lists all possible input combinations in an ordered fashion.
- that the symbol "~" is used for logic negation.
- that the symbol "&" is used for logic ANDing.
- that the symbol "|" is used for logic ORing.

Finite State Machines (FSMs)

- FSMs are the "work horse" in digital systems.
- FSMs can be implemented with hardware.
- We look at "synchronous" FSMs only:
 The "clock signal" controls the action over time
- FSMs can be described with three main "views":
 - The functional view with the "state diagram"
 - The timing view with the "timing diagram"
 - The structural view with the "logic circuit diagram"

Time flows continuously

time

We cut time into slices



Time slices are strictly ordered



Clock signal



We use the clock signal ("clk") in order to advance time from slice to slice.

Rising clock edge



With **rising clock edges** we define the transition between neighboring time slices. The negative clock edges have no importance.

Clock period



Synchronous finite state machine (= automaton)



A **synchronous FSM** is clocked by a clock signal ("clk"). In each clock period, the machine is in a defined (current) **state**. With each rising edge of the clock signal, the machine advances to a defined next state.

We are interested in "finite state machines" (FSMs). FSMs have only a finite number of states. The sequence of states can be defined in a state diagram.



State diagram:

We denote the states with circles and give them **symbolic names**, e.g. A, B, and C.





С

State diagram:

We define one of the states as the initial state.



B

С

In the beginning...



Initially, i.e. shortly after switching on the FSM and before the first rising edge of clock, there is the initial period. In this period, the FSM is in the "initial state". 15



Rechnerorganisation, Stefan Mangard 2019, Slides by KC Posch

C

The sequence of states can also be defined in a state transition table.

current	next
state	state
Α	В
В	С
C	A





Initially, the FSM is in the initial state A.

With every positive clock edge, the next state becomes the current state.

An FSM has **always** a next state.

In order to technically realize ("implement") a state diagram, we start by giving each state a unique number.



Popular state-encoding schemes



- Binary encoding
 - needs minimum amount of flip-flops.

- One-hot encoding
 - Tends to have a simpler next-state logic.



Binary encoding

state	number
А	00
В	01
С	10
	21

For a Verilog example of this FSM see

chapter_03/example_00/01_fsm_moore_no_input.v

Binary encoding: The state transition table has also only binary numbers.

00

10

current	next
state	state
00	01
01	10
10	00

We also enter the unused combination "11". This state does not exist. "x" stands for "Don't care".



current	next	
state	state	
00	01	
01	10	
10	00	
11	XX	

We define names for the two state bits, e.g. s1, s0.



curr	ent	next
s1	s0	s1 s0
0	0	0 1
0	1	1 0
1	0	0 0
1	1	x x

Each state bit is stored in a flipflop



A flipflop stores a new value, when a rising edge occurs on the clock input



The flipflop stores the value, which it sees on its input.



Summary: A D-flip-flop samples its input value D and stores this value when a rising edge of clk occurs



Thus, a D-type flipflop can be seen as a 1-bit photo camera. Rechnerorganisation, Stefan Mangard 2019, Slides by KC Posch

In our example, we need 2 flip-flops for storing the state bits.



Flipflops which can store several bits are also called "registers".

We use the state transition table as a "lookup table"...



...and thus find out the next state



At each positive edge of clk, the next state gets stored as the current state.



In order to get the initial state "00", we use flipflops with an "asynchronous reset input". Shortly after switching on the circuit, we apply "areset".



The state transition function in this example can be derived from the truth table: next s0 = (~s1) & (~s0)



The state transition function: next s0 = (~s1) & (~s0) next s1 = (~s1) & s0


Structural diagram of the FSM: State-transition function, storage elements, and feedback of state.



Essence so far

State diagram:

"next state" is a function of (current) "state"

State transitions:

the "next state" becomes (the current) "state" on the rising edge of the clock

The FSM modeled and simulated with Logisim



Inputs

FSMs can also have **inputs** influencing the transition to the next state

In this example we see that the one-bit input "in" influences the choice of the state after B.

next state = f(state, input)



FSMs can also have **inputs** influencing the transition to the next state

next state = f(state, input)

In this example we see that the one-bit input "in" influences the choice of the state after B.

The following state can also be the same as the current state. Rechnerorganisation, Stefan Mangard 2019, Slides by KC Posch



The state transition table

current state	in	next state
A	0	B
A	1	B
B	0	A
B	1	C
C	0	A
C	1	C



For a Verilog example of this FSM see

chapter_03/example_00/01_fsm_moore_no_output.v



For a timing diagram, we need to choose values for the input signal "in". Only after some choice for "in" we can derive the sequence of states from the state diagram.



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For a timing diagram, we need to choose values for the input signal "in". Only after some choice for "in" we can derive the sequence of states from the state diagram.

Binary state encoding: Instead of symbolic state names we use numbers

current in state		next state		
0 0	0 0	0 1	0 0	1
0	1	0	0	0
1	<u>г</u> О	1 0		0
1	0	1	1	0



"11" does not exist: We use "Don't Care" as the following state

current in state		next stat	t e	
0	0	0	0	1
0	0	1	0	1
0	1	0	0	0
0	1	1	1	0
1	0	0	0	0
1	0	1	1	0
1	1	0	X	X
1	1	1	X	X



I have ordered the lines in the state transition table from 0 to 7. This makes it easier to "read" the table.



We call the state bits "s1" and "s0"

	cu s1	rrent s0	in	next s1 s0	
)	0	0	0	0	1
L	0	0	1	0	1
2	0	1	0	0	0
3	0	1	1	1	0
ł	1	0	0	0	0
5	1	0	1	1	0
5	1	1	0	x	x
7	1	1	1	x	x



next s0 = $((\sim s1) \& (\sim s0) \& (\sim in))$ | $((\sim s1) \& (\sim s0) \& in)$

	cu	rrent	in	ne	xt
	s1	s0		s1	s0
)	0	0	0	0	1
L	0	0	1	0	1
2	0	1	0	0	0
3	0	1	1	1	0
1	1	0	0	0	0
5	1	0	1	1	0
5	1	1	0	Х	0
7	1	1	1	X	0



next s1 = $((\sim s1) \& s0 \& in)$ | $(s1 \& (\sim s0) \& in)$

	cu s1	rrent s0	in	ne s1	xt s0	
)	0	0	0	0	1	
L	0	0	1	0	1	
2	0	1	0	0	0	
3	0	1	1	1	0	
1	1	0	0	0	0	
5	1	0	1	1	0	
5	1	1	0	0	0	
7	1	1	1	0	0	



Structural diagram of the FSM



Implementation with Logisim



Outputs

FSMs typically also have outputs

In this example we see that the outputs are a function of the state. We write the output values into the circles.

We call such machines also "Moore machines":

output = f(state)



We define the outputs with the "output function"

Moore machines:

output = f(state)

state	output
А	4
В	3
С	2





For a timing diagram, we need to choose values for the input signal "in". Only after some choice for "in" we can derive the sequence of states from the state diagram.

Moore machines:

output = f(state)





Moore machines:

output = f(state)

state	02 01 00			
А	1	0	0	
В	0	1	1	
С	0	1	0	



For a Verilog example of this FSM see

chapter_03/example_00/01_fsm_moore_with_output_function.v

Binary state encoding: We define the outputs with binary values



state		02 01 00		
0	0	1	0	0
0	1	0	1	1
1	0	0	1	0









Structural diagram of the FSM



Implementation with Logisim



Essence of Moore Machines the state is stored in a register



Essence of Moore Machines the state is stored in a register



Essence of Moore Machines the state is stored in a register



Essence of Moore Machines With the next-state function f we compute the next state: next state = f(state, input)


Essence of Moore Machines

With the output function we compute the output values:

output = g(state) next s0 = (~s1 & ~s0 & ~in) o2 = ~s1 & ~s0 02 s0 | (~s1 & ~s0 & **D0 Q0** o1 = (~s1 & s0)in) > 01 s1 | (s1 & ~s0) D1 **Q1** next s1 = (~s1 & s0 & in) 00 in 00 = ~s1 & s0|(s1 & ~s0 & in)|areset clk

Essence of Moore Machines



There exist 2 types of machines: check out the LITTLE but IMPORTANT difference

- Moore Machines
 - next state = function of state and input
 - output = function of state
- Mealy Machines
 - next state = function of state and input
 - output = function of state and input

Essence of **Moore** Machines



Essence of Mealy Machines

output = g(state, input)



An example for a Mealy Machine

We write the **output values** next to the transition arrows, since the output depends not only on the state, but can also depend on the input.



The state transitions are the same as in the previous example with the Moore Machine

current state	in	next state
A	0	B
A	1	B
B	0	A
B	1	C
C	0	A
C	1	C



The output function

The output function can be derived from the state diagram. output = g(state, input)

state	in	output
Α	0	4
A	1	4
В	0	3
В	1	1
C	0	2
C	1	0



Timing diagram



For a timing diagram, we need to choose values for the input signal "in". Only after some choice for "in" we can derive the sequence of states from the state diagram. We see here also, how the value of "in" immediately influences the value of "out". Rechnerorganisation, Stefan Mangard 2019, Slides by KC Posch

For a Verilog example of this FSM see

chapter_03/example_00/04_fsm_mealy.v

Binary encoding: Re-writing the output table with binary values.

And completing the table with the unused bit combinations.



We can derive the logic functions for o2, o1, and o0

s1	s0	in	o2	o1	00
0	0	0	1	0	0
0	0	1	1	0	0
0	1	0	0	1	1
0	1	1	0	0	1
1	0	0	0	1	0
1	0	1	0	0	0
1	1	0	x	Х	X
1	1	1	X	Х	X

o2 = ~s1 & ~s0

We can derive the logic functions for o2, o1, and o0

s1	s0	in	о2	o1	00
0	0	0	1	0	0
0	0	1	1	0	0
0	1	0	0	1	1
0	1	1	0	0	1
1	0	0	0	1	0
1	0	1	0	0	0
1	1	0	x	Х	X
1	1	1	X	Х	X

o2 = ~s1 & ~s0

```
o1 = (~s1 & s0 & ~in)
| (s1 & ~s0 & ~in)
```

We can derive the logic functions for o2, o1, and o0

s1	s0	in	о2	o1	00
0	0	0	1	0	0
0	0	1	1	0	0
0	1	0	0	1	1
0	1	1	0	0	1
1	0	0	0	1	0
1	0	1	0	0	0
1	1	0	x	Х	X
1	1	1	X	Х	X

o2 = ~s1 & ~s0

o1 = (~s1 & s0 & ~in) | (s1 & ~s0 & ~in)

o0 = ~s1 & s0

The result



Modeling with Logisim



We can combine machines

- Combining Moore Machines causes no problem. We get another Moore Machine.
- Combining a Moore Machine with a Mealy Machine causes also no problem. We get a Moore Machine or a Mealy Machine.
- Combining two Mealy Machines can cause troubles: One needs to avoid combinational loops!

The combination of two Moore Machines creates again a (more complex) Moore Machine



90

We can even connect More Machines in a loop-like fashion



The combination of a Moore Machine with a Mealy Machine creates a Moore Machine or a Mealy Machine



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The combination of a Moore Machine with a Mealy Machine creates a Moore Machine or a Mealy Machine



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The combination of two Mealy Machines is "dangerous": You need to avoid "combinational loops"



The combination of two Mealy Machines is "dangerous": You need to avoid "combinational loops"



Summary

- All digital logic can in principle be built with Moore Machines and Mealy Machines.
- You always start by defining the function with a state diagram.
- If you choose values for the input signal(s), then you can derive the timing diagram by using the state diagram.
- From a state diagram, you can always derive a circuit diagram.

Algorithmic State Machines

Algorithmic State Machines (ASMs)

ASMs are a useful extension to finite state machines

 ASMs allow to specify a system consisting of a data path together with its control logic

 All FSM state diagrams have an equivalent ASM diagram







Register-Transfer Statements

- Register-transfer statements define the change of a value stored in a register.
- Values in registers can only change at the active (= rising) edge of clock.
- We denote "register-transfer statements" with a "left arrow" (" \leftarrow ")
- Example: "a ← x" means that the value in the register "a" gets the value of "x" at the "next" active (= rising) edge of clock.
- We can specify register-transfer statement in an ASM diagram.

ASM diagram with two register-transfer statements





"=" versus "←"

- With the equal sign ("=") we denote that the output of the FSM has a certain value during a particular state.
- With the left-arrow ("←") we denote a register-transfer statement: The register value left of the arrow changes to whatever is defined right of the arrow upon the next active (= rising) edge of clock.

Several register-transfer statements can be specified within one state



The values stored in register X and register Y become 0 at the state transition from state A to state B.

The value in register X does not change upon leaving state B. The value stored in register Y gets the value of register X upon leaving state B.

The value in register X gets incremented at the state transition following state C.

Several register-transfer statements can be specified within one state



The values stored in register X and register Y become 0 at the state transition from state A to state B.

The value in register X does not change upon leaving state B. The value stored in register Y gets the value of register X upon leaving state B.

The value in register X gets incremented at the state transition following state C. Register Y gets the "old" value from X; i.e the value before X gets incremented.

For a Verilog example of this ASM graph see

chapter_03/example_00/05_asm.v
Manual Synthesis of an ASM Graph in Logisim

Register-transfer statements define the data path



The "neighborhood" of register X:

 $\begin{array}{c} X \leftarrow 0 \\ X \leftarrow X \\ X \leftarrow X + 1 \end{array}$

The "neighborhood" of register Y:

 $\begin{array}{c} Y \leftarrow 0 \\ Y \leftarrow X \end{array}$

The neighborhood of register X

- Case 0: $X \leftarrow X$
- Case 1: $X \leftarrow X+1$
- Case 2: $X \leftarrow 0$

We need to distinguish between 3 cases.

The neighborhood of register X

clrx	incx	action
0	0	$x \leftarrow x$
0	1	X ← X+ 1
1	0	X ← 0

We use binary notation and name the two binary select variables.

The neighborhood of register X

clrx	incx	action
0	0	$X \leftarrow X$
0	1	X ← X+ 1
1	0	X ← 0

We model the truth table with a multiplexer. For incrementing X we use an adder.

With Logisim we can model the neighborhood of Register and also simulate.



The neighborhood of register Y

In a similar way, we can model the neighborhood of Y.



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clry

 \mathbf{O}

()

1

ldy

()

action

 $Y \leftarrow Y$

 $Y \leftarrow X$

The datapath



We combine the two neighborhoods.

Note that both neighborhoods are Moore machines.

The Moore machine for X has 2 inputs: clrx and incx. Since we have chosen an 8-bit register for X, we have 256 possible states. The output function is the identity function.

The connection of the two is again a Moore machine. Thus, The datapath is a Moore machine.

Register-transfer statements define the data path



The control logic needs to provide the control signals



Truth table of output logic of controller





state	clrx	incx	clry	ldy	out	
A	1	0	1	0	4	
B	0	0	0	1	3	
C	0	1	0	1	2	

Truth table of output logic of controller



Truth table of next-state logic of controller



sta	ite	clrx	incx	clry	ldy	out
0	0	1	0	1	0	100
0	1	0	0	0	1	011
1	0	0	1	0	1	010

state	in	next_state
А	0	В
A	1	В
В	0	А
В	1	С
C	0	А
С	1	А

Truth table of next-state logic of controller



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ldy

out

Truth table of next-state logic of controller



122

From truth table to implementation



s1 s0	clrx	incx	clry	ldy	out
0 0	1	0	1	0	100
01	0	0	0	1	011
10	0	1	0	1	010
11	Х	Х	Х	Х	XXX

s1	s0	in	ns1 ns0
0	0	0	0 1
0	0	1	01
0	1	0	0 0
0	1	1	1 0
1	0	0	0 0
1	0	1	0 0
1	1	0	хх
1	1	1	ХХ

The controller and the data path



That's it.

- In principle, we can describe any synchronous automaton with an ASM diagram.
- In principle, every synchronous digital system can be described by a collection of ASM diagrams.
- The transformation from an ASM diagram into a logic circuit is defined by an algorithm; we call this algorithm "synthesis".

Hands-On Example

City of Tiny Lights



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Overview

 Design a finite state machine from a functional description to logic layer.

• Decompose an ASM-diagram into data path and control logic.

• Connect several finite state machines into a larger finite state machine.

A traffic light

• Usually green for cars and red for pedestrians.

- A pedestrian wants to cross.
- The pedestrian hits a button and it gets green for her.

• Shortly afterwards, the cars get green light again.

An ASM-diagram of a simple traffic light

Pedestrians have access to an input button to ask for green light.



An ASM-diagram of a simple traffic light

Pedestrians have access to an input button to ask for green light.

Input button is called "ped".



An ASM-diagram of a simple traffic light

outC = 001 outP = **1**00

outC is output visible to cars

outP is output visible to pedestrians

"ped" is an input button for pedestrians to request green light











Next-state table (Logisim)

s2	s1	s 0	ped	ns2	ns1	ns0
0	0	0	0	0	0	0
0	0	0	1	0	0	1
0	0	1	0	0	1	0
0	0	1	1	0	1	0
0	1	0	0	0	1	1
0	1	0	1	0	1	1
0	1	1	0	1	0	0
0	1	1	1	1	0	0
1	0	0	0	1	0	1
1	0	0	1	1	0	1
1	0	1	0	0	0	0
1	0	1	1	0	0	0
1	1	0	0	x	х	х
1	1	0	1	x	х	х
1	1	1	0	X	х	х
1	1	1	1	X	х	х



Synthesis mit Logisim

			Со	mbir	ation	al Ana	alysis		
	Inputs	Out	puts	Та	ıble	Exp	ressi	on	Minimized
		s2	s1	s 0	ped	ns2	ns1	ns0	
		0	0	0	0	0	0	0	
		0	0	1	0	0	1	0	
		ŏ	ŏ	1	1	ŏ	1	ŏ	
10		^		B	uild C	ircuit			
	\rightarrow	Circ	Destination Project: traffic_light_2015 ‡ Circuit Name: nsl1 Use Two-Input Gates Only Use NAND Gates Only						
				Bu	ild Ci	ircuit			



Next-state function synthesized in Logisim





Output table for cars

s2	s1	s0	red	yellow	green
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	1	0	0
0	1	1	1	0	0
1	0	0	1	0	0
1	0	1	0	1	0
1	1	0	x	х	х
1	1	1	x	х	x







Output table for ped's

s2	s1	s 0	red	yellow	green
0	0	0	1	0	0
0	0	1	1	0	0
0	1	0	0	1	0
0	1	1	0	0	1
1	0	0	0	1	0
1	0	1	1	0	0
1	1	0	х	х	х
1	1	1	х	х	х







Traffic light (part 1)



The problem

- A nasty pedestrian sticks a stick into the "ped"-button and leaves.
- Thus, the cars are always held by red, although there is no pedestrian.
- We want to avoid this "denial-of-service" attack.
- We add a second algorithmic state machine which tries to take care of this problem: After a pedestrian has hit the button and got green, there is a time delay before it can get green again.



Pedestrians hit "button"

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1010

zea0



zea0

9-

MUX

ld9

0

decr

ղ



ped= 0



Data path is a Moore machine







state	butt	zeq0	next_state
А	0	х	А
А	1	х	В
В	X	x	С
С	X	x	D
D	X	0	С
D	X	1	А

"x" stands for "don't care"

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t1 t0	butt z	eq0	nt1	nt0
00	0	х	0	0
0 0	1	х	0	1
0 1	Х	х	1	0
1 0	Х	х	1	1
1 1	Х	0	1	0
1 1	Х	1	0	0

"x" stands for "don't care"

	t1	t0	butt	zeq0	nt1	nt0
_	0	0	0	0	0	0
В	0	0	0	1	0	0
	0	0	1	0	0	1
	0	0	1	1	0	1
	0	1	0	0	1	0
	0	1	0	1	1	0
	0	1	1	0	1	0
	0	1	1	1	1	0
С	1	0	0	0	1	1
	1	0	0	1	1	1
	1	0	1	0	1	1
D	1	0	1	1	1	1
	1	1	0	0	1	0
	1	1	0	1	0	0
	1	1	1	0	1	0
	1	1	1	1	0	0
)						

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ped= 0

ld9 = 0

decr = 0

butt

ped = 1

ld9 = 1

decr = 0

ped = 0

ld9 = 0

decr = 1

ped = 0

ld9 = 0

decr = 0

zeq0

1

0

Α







Output logic

t1 t0	ld9	decr	ped
00	0	0	0
01	1	0	1
1 0	0	1	0
1 1	0	0	0



Output logic

t1 t0	ld9	decr	ped
00	0	0	0
01	1	0	1
1 0	0	1	0
1 1	0	0	0



Output logic











City of tiny lights



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