63. For the following formula in the Theory of Uninterpreted Functions and Equality $\mathcal{T}_{UE}$, use Ackermann's reduction to find an equisatisfiable formula in the Theory of Equality $\mathcal{T}_E$.

(a) [1 Point] $f(x) = g(x) \lor z = f(y) \rightarrow f(z) \neq g(y) \land x = z$

(b) [2 Points] $f(f(g(x))) = g(g(f(x)))$

(c) [2 Points] $f(a, b) = x \land f(x, y) \neq g(a) \lor f(m, n) = b$

64. For the following formulas in $\mathcal{T}_E$, use the graph-based reduction to find equisatisfiable propositional formulas.

(a) [1 Point] $u \neq z \lor x = u \rightarrow x = y \land y \neq z$

(b) [1 Point] $a = b \lor b \neq c \rightarrow \neg(c \neq d \lor d = e \land e = f)$

(c) [2 Points] $a \neq b \land b = c \lor c = d \rightarrow \neg(d \neq e \lor e = f) \land \neg(f = g \land a \neq e)$

65. [2 Points] Use the Congruence Closure algorithm to determine whether or not the following formula from the conjunctive fragment of $\mathcal{T}_{UE}$ is satisfiable:

\[
\begin{align*}
  a &= b \land f(c) = b \land c = d \land e \neq a \land f(a) \neq f(e) \land \\
  f(e) &= f(b) \land f(d) = f(a) \land f(b) \neq f(d) \land f(c) = c
\end{align*}
\]

66. [4 Points] Use lazy encoding to check if the following formula from $\mathcal{T}_{UE}$ is satisfiable. Perform DPLL to obtain a full assignment for the equalities which you can then check with the Congruence-Closure algorithm. If you find a theory conflict, obtain a blocking clause
from the Congruence-Closure algorithm, restart the DPLL algorithm at decision level 0 with the new blocking clause added. For simplicity, do not use conflict-driven clause learning. I.e., the only clauses you are supposed to add are blocking clauses coming from Congruence Closure.

In each step of the DPLL algorithm, first try to perform Boolean Constraint Propagation (BCP), if applicable, then try to set Pure Literals (if applicable). If neither BCP nor Pure Literals are applicable, make a decision. When backtracking, always undo exactly one decision. (Note that undoing one decision implies undoing all BCP- and Pure-Literal-based assignments that were done because of this decision.)

**Important:** If you have to make a decision in DPLL, use the following order of literals to select a theory literal to assign a truth value to.

1. \( a = b \)
2. \( x = y \)
3. \( f(x) = f(y) \)
4. \( f(a) = f(b) \)

Always assign \textit{false} to the literal first, and \textit{true} second.

Formula to check: \( T_{UE} := \)

\[
(a = b \lor x = y) \land \\
(a \neq b \lor x \neq y) \land \\
(f(a) = f(b) \lor f(x) \neq f(y)) \land \\
(f(a) \neq f(b) \lor f(x) = f(y))
\]